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**Spécialité :** *Electronique*

**Option :** *Signal et Télécommunications*

## *Thème*

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# A Genetic Approach For Optimal Registration in Medical and Satellite Imaging

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# ABSTRACT

*Image registration is a fundamental task used in image processing to match two or more images taken at different times, from different sensors or from different viewpoints. It has found applications in numerous real-life applications such as remote sensing, medical image analysis, computer vision and pattern recognition. The objective is to find in a huge search space of geometric transformations, an acceptable accurate solution in a reasonable time to provide better registered images.*

*Over the years, a broad range of techniques has been developed for various types of data and problems. Exhaustive search is computationally expensive and the computational cost increases exponentially with the number of transformation parameters and the size of the data set. In this work, we present an efficient image registration algorithm that uses genetic algorithms within a multi-resolution framework based on the Non-Subsampled Contourlet Transform (NSCT). An adaptable genetic algorithm for registration is adopted in order to minimize the search space. This approach is used within a hybrid scheme applying the two techniques, fitness sharing and elitism. Two NSCT based methods are proposed for registration. A comparative study is established between these methods and a wavelet based one. Because the NSCT is a shift-invariant multidirectional transform, the second method is adopted for its search speeding up property. Simulation results clearly show that both proposed techniques are really promising methods for image registration compared to the wavelet approach, while the second technique has led to the best performance results of all. Moreover, to demonstrate the effectiveness of these methods, these registration techniques have been successfully applied to register SPOT, IKONOS and Synthetic Aperture Radar (SAR) images. The algorithm has been shown to work perfectly well for multi-temporal satellite images as well, even in the presence of noise.*

*In another contribution, we have developed a 2D point set registration based on genetic algorithms, in which a feature based image registration method between two data sets is performed using genetic algorithms. The procedure focused on aligning two data sets through a robust method in order to obtain the best correspondence between points. The registration optimization problem is also solved by a Genetic algorithm method. The results show the efficiency of this method for registration of satellite images as well as medical images.*

**Keywords:** *genetic algorithms, image registration, multi-resolution analysis, nonsubsampled contourlet transform, wavelet transform, ICP algorithm, 2D point set registration.*

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# List of Abbreviations

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<b>GAs</b>	<i>Genetic Algorithms</i>
<b>CT</b>	<i>Computer Tomography</i>
<b>MRI</b>	<i>Magnetic Resonance Imaging</i>
<b>SPOT</b>	<i>Satellite Pour l'Observation de la Terre</i>
<b>SAR</b>	<i>Synthetic Aperture Radar</i>
<b>DFB</b>	<i>Directional Filter Bank</i>
<b>PDFB</b>	<i>Pyramidal Directional Filter Bank</i>
<b>NSDFB</b>	<i>Nonsampled Directional Filter Bank</i>
<b>NSCT</b>	<i>Nonsampled Contourlet Transform</i>
<b>ICP</b>	<i>Iterative Closest Point</i>

# INTRODUCTION

## INTRODUCTION

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*A key issue in many application areas as computer vision, remote sensing, medical imaging, pattern recognition is to deal with images that are acquired by imperfect imaging systems. Different image acquisition conditions, such as imaging geometry, sensing/environmental conditions and sensor errors, may introduce several types of distortions and degradations to the observed image compared to the original scene. This may result in images that suffer degradations such as contamination with noise, blurring and brightness/contrast change. Moreover, image planes may have different orientations, scales, positions or may have undergone geometric distortions.*

*One of the important applications that concerns geometric distortions of images is "image registration". Given two or more images to be registered, an image registration system estimates the parameters of the geometric transformation model that maps a given target image, that was taken from different viewing positions or at different times to the reference one. Registration methods are increasingly in demand by many image analysis and processing systems, as the first step, to accurately capture the geometric transformations of image data. For example, image registration is used in image stitching, where multiple images are combined to produce a panorama or larger images. Image registration is used in the analysis of remotely sensed data (Bentoutou and al., 2005) where an image must be transformed, using image registration techniques, to match the orientation and the scale of previously acquired images. Also, in motion analysis applications (Bentoutou & Taleb, 2005) (Bentoutou and al., 2002), image registration is utilized to find changes between subsequent frames in a video sequence and in medical imaging systems. Images need to be aligned first which can be accomplished through a registration process, before they can be compared for analysis and diagnostic purposes.*

*In general, its applications can be divided into three main groups according to the manner of the image acquisition:*

- 1. Different times: Images of the same scene are acquired at different times. The aim is to find and evaluate changes in the scene. Examples of applications are landscape planning in remote sensing, automatic change detection in video surveillance, motion tracking in computer vision, monitoring of healing therapy and tumor evolution in medical imaging, and motion estimation and super resolution reconstruction in video processing.*

## INTRODUCTION

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2. *Different viewpoints: Images of the same scene are acquired from different viewpoints. The aim is to gain a larger 2-D view or a 3-D representation of the scene being imaged. Examples of applications are image mosaicking of the surveyed area in remote sensing, shape recovery and structure from motion in computer vision, and sprite generation and coding in video compression.*
3. *Different sensors: Images of the same scene are acquired from different sensors. The aim is to integrate the information obtained from different sensors to gain more complex and detailed representation of the scene. Examples of applications are multisensor image fusion in remote sensing and medical imaging, monitoring activities in multisensor surveillance, and image fusion in vehicular navigation.*

*Image registration can be regarded as an optimization problem, where the goal is to find the best transformation parameters which maximize the measure similarity between compared images. Recently, Genetic Algorithms (GAs) have received much attention as they are used as a tool for searching the large, poorly understood spaces that arise in many application areas of science and engineering due to their robustness.*

*The main specificity of the GA as an optimization method is their implicit parallelism, which is a result of the evolution and hereditary process. A GA is, in fact, a driven stochastic search technique, which combines stochastic (represented by mutation operator) and “logical” search (represented by crossover of parental chromosomes and survival of the fittest by appropriate selection). These characteristics of GAs offer possibilities for their improvement by appropriate balance between exploration and exploitation.*

*Genetic Algorithms are good at taking large, potentially huge search spaces and navigating them, looking for optimal combinations of things, the solutions, one might not otherwise find in a lifetime.”*

Salvatore Mangano, Computer Design, May 1995

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*GA was introduced by Holland in the 1975s as an algorithmic concept based on the Darwinian-type survival-of-the-fittest strategy with sexual reproduction. Just like the animals and plants on the earth, the stronger individuals in the population have high chance of having an offspring. A GA is used as a computerized searching and optimization procedure that uses principles of nature genetics and nature selection. The basic approach is to model the possible solutions in the search problem. In general, if a problem solving mechanism can be represented in a reasonably compact form, the GA techniques can be applied by using procedures to maintain a population of knowledge structure that represents candidate solutions, and then let that population evolve over time through competition.*

*According to the natural selection theory, individuals that are better fit to a given environment are more likely to survive. GAs use the same tools, points in the search space are encoded to form a population, and a selection process chooses high-fitness strings according to a certain criterion. This means that strings with higher fitness values have a higher probability of reproducing new strings in the new generation. A GA uses a fitness function to determine the performance of each chromosome. Therefore, the fitness function should measure the registration quality matching. Two operators: crossover and mutation introduce new individuals into the population. These operations enable GAs to use a relatively few samples to search large spaces.*

### ❖ *Thesis contributions*

*A lot of researchers have attempted to apply GAs in image registration that can take advantage of the robustness of GAs in finding the optimum transformation. Although, most GA applications have been hybrids (Lobo & Goldberg 1997). This happens because there is a possibility of incorporating domain knowledge which gives it an advantage over a pure blind search. Hybridization of GAs has been done on a range of image registration applications (Silva and al., 2003), (Mashohor & Arslan, 2006). This technique has been proved to provide fast convergence and good performance in finding correct registrations compared to a GA alone.*

*The hybrid method of cooperating GAs with elitism and fitness sharing techniques is proposed in this thesis to maximise the robustness of the search technique in order to provide better quality in image registration.*

## INTRODUCTION

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*The registration problem typically comes down to determining the matching transformation parameters by traversing the search space looking for the maximum of the similarity metric. Different search strategies can be applied and combined as well, in order to improve the accuracy results and decrease the costs of the registration method. A common approach for improving the efficiency of image registration is to apply a multi-resolution framework.*

*As observed by Zavorin and Lemoigne (Zavorin & LeMoigne, 2005), there are a number of advantages using a multiresolution approach compared to working solely with the original images. It can reduce computation time by performing much of the work at coarse resolutions, leaving minor adjustments to later stages. Since this type of image decomposition usually involves low-frequency smoothing, this regularizes the registration problem thus yielding better convergence properties and improved accuracy of the search algorithm. Finally, if image scales differ significantly, decomposition could be used to bring the two images into similar scales, which may be advantageous for some registration algorithms.*

*The basic idea of use the multi-resolution property is to produce coarse to fine representations of the input and reference images. At the beginning of the search procedure, the transformation parameters are searched for in a coarse manner using simple search strategy. The results are then refined by an accurate, though more complex, search strategy.*

*The wavelet transform presents a powerful tool for multi-resolution analysis. Wavelets can be used to improve the image registration accuracy by considering both spatial and spectral information and by providing multi-resolution representation to avoid losing any global or local information (LeMoigne, 1994), (Corvi & Nicchiotti, 1995). However, this technique suffers from poor directional selectivity and not having the shift-invariant property due to its downsampling stage. Recently, many multi-resolution multi-directional approaches are adopted such as the contourlets and his shift invariant transform named nonsubsampling contourlets transform.*

*Therefore, the main topic of this thesis is to presents a rigid registration technique that uses the genetic algorithms as an optimization method applied within a multi-resolution framework based nonsubsampling contourlet transform which combine local search methods with global ones balancing exploration and exploitation, to speed up the search of the best transformation parameters.*



## INTRODUCTION

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*Our choice of algorithms is primarily dictated by the following criteria: applicability for registering remotely sensed images, shown good results on satellite images, and good potential for registering multi-temporal images even with presence of noise. Experimental results show that the proposed approach is a promising method compared to the wavelet transform for registration of satellite images.*

*In another contribution, we have developed a novel approach of a registration method based on feature points and GAs. Consequently, feature-based approaches have the advantage of greatly reducing the computational complexity. Depending on the feature extraction process, these approaches may also be more robust to intensity variations that arise during, for example, cross modality registration. Also, features may be chosen to help reduce sensor noise. In this context, our aim is to explore this field of point set registration especially for medical application.*

*Point-based registration involves the determination of the co-ordinates of corresponding points in different images and the estimation of the geometrical transformation using these corresponding points. Then, the key of image registration is to find the proper transformation of one image to another image so that each point of one image is spatially aligned with its corresponding point of the other. It is widely used in areas such as range data fusion, medical image alignment, object localization, tracking, and object recognition. The best known iterative method for aligning a set of data is Iterative Closest Point (ICP) introduced by Besl and McKay (Besl & McKay,1992) which is based on the search of pairs of nearest points in the two data sets, and estimating the rigid transformation, which aligns them. This algorithm is composed of two basic procedures. The first one is to find matching points, and the second one is to estimate the transformations iteratively for these points until some stop distance criteria is satisfied.*

*In this context, a feature based image registration method between two data sets is performed using genetic algorithms. Our approach can be seen as an improvement to the state of the art as it combines the positive aspects of the different already well studied methods such as point registration and the application of GAs. The procedure focused on aligning two data sets through a robust method in order to obtain the best correspondence between points. This correspondence set will allow us to calculate a rigid transformation that registers*

*the images accurately based on nearest neighbor criteria to calculate the distance. The registration optimization problem is solved by the GAs method. The results show the efficiency of this method for registration of satellite images as well as medical images.*

### ❖ *Thesis Organization*

*This thesis contains two parts englobing six chapters. Following the introduction, the thesis is organized as follows:*

*Part A includes the state of the art chapters: The first chapter gives an overview of image registration. This includes image registration, classification and applications of image registration. The second chapter introduces the basis of genetic algorithms and gives a comprehensive overview of this class of methods of optimization and their applications in image registration. The different multi-resolution approaches are presented in the third chapter.*

*Part B is essentially the contributions chapters. A first contribution is a proposed registration algorithm based on a genetic algorithm as an optimization method combined with a multi-resolution approach based on the wavelet transform in order to refine the transformation parameters and is presented in detail in the fourth chapter with the experimental results. Another contribution of this thesis is presented in fifth chapter in which we describe the use of the nonsubsampling contourlet transform for image registration based on genetic algorithms. Finally, the sixth chapter presents our last contribution in image registration based on a point set method using genetic algorithms.*

*At last, we finalize this thesis with a conclusion.*

# **PART A**

## **STATE OF THE ART CHAPTERS**

# CHAPTER I

## **IMAGE REGISTRATION THEORY**

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## **I.1 Introduction**

Image registration is a classical problem in several image processing applications (*Taleb and al., 2001*), (*Bentoutou & Taleb, 2005A*), (*Bentoutou and al., 2005*) where it is necessary to match two or more images of the same scene. Some examples of applications are (*Fanseca & Manjunath, 1996*):

- *Integration of information taken from different sensors*

In remote sensing, a great number of sensors for global monitoring are available; each of them with different spectral, spatial and radiometric characteristics. It is useful to combine and analyze the image data to take advantage of their characteristics and improve the information extraction process. For example, the combination of images obtained from SPOT and Landsat Thematic Mapper (TM) satellites has been used in applications such as monitoring urban growth. SPOT images present better spatial resolution while the TM images have better multispectral resolution. The intensity Hue saturation transform can be used to merge the SPOT panchromatic band with TM multispectral bands and generate another color enhanced image with high spatial resolution. The alignment of the images is the first step in this data transformation.

- *Analysis of changes in images taken at different times*

In multitemporal image analysis, the objective is to detect changes which have occurred over a certain time period. A simple method to find changes in a pair of images is to overlay the images and detect the differences between them. Because these images are taken at different times and under different conditions, they have to be aligned prior to comparative processing.

- *In computer vision, registration is necessary in extracting structure from motion and object recognition.*

Other problems such as finding cloud heights, satellite image composite generation, weather prediction, and wind direction measurements also involve the registration process.

For typical image registration problems, the sources of differences between two images fall into four categories (*Morgan, 1998*):

1. Differences of alignment between images are caused by a spatial mapping from one image to the other. Typical mappings involve translation, rotation, warping, and scaling. For infinite continuous domain images, these differences are a result of a spatial mapping from one image to the other. Changing the orientation or parameters of the imaging sensor can cause differences of alignment.
2. Differences from occlusion occur when part of a finite image moves out of the image frame or new data enters the image frame of a finite image due to an alignment difference, when sensor errors produce identifiably invalid data in an image, or when an obstruction comes between the imaging sensor and the object being imaged. For example, in satellite images, clouds frequently occlude the earth.
3. Differences from noise occur from sampling error and background noise in the sensor, and from unidentifiably invalid data introduced by sensor error.
4. Differences due to change are actual differences between the objects or scenes being imaged. In satellite images, lighting, erosion, construction, and deforestation are examples of differences due to change. It may be impossible to distinguish between change and noise.

## **I.2 Elements of Image Registration**

The goal of registration is to establish geometric correspondence between the images so they may be transformed, compared, and analyzed in a common reference frame. Due to the diversity of images to be registered and due to the various types of degradations, it is impossible to design a universal method applicable to all registration tasks. Every method should take into account not only the assumed type of geometric deformation between the images but also radiometric deformations and noise corruption, required registration accuracy and application dependent data characteristics (*Zitova & Flusser, 2003*).

Nevertheless, the majority of the registration methods consist of the following four steps:

- **Feature detection.** Identifies a set of relevant features in the two images, such as edges, intersections of lines, region contours, etc. Salient and distinctive objects (closed boundary regions, edges, contours, line intersections, corners, etc...). For further processing, these features can be represented by their point representatives called control points.
- **Feature matching.** In this step the correspondence between the features detected in the sensed image and those detected in the reference image is established. Various feature descriptors and similarity measures along with spatial relationships among the features are used for the purpose.
- **Transform model estimation.** The type and parameters of the so-called *mapping functions*, aligning the sensed image with the reference image are estimated. The parameters of the mapping functions are computed by means of the established feature correspondence.
- **Image resampling and transformation.** The sensed image is transformed by means of the mapping functions. Image values coordinates are estimated by an appropriate interpolation technique. The nearest-neighbour or bilinear interpolation is sufficient in most cases.

The feature space extracts the information in the image that will be used for matching. The search space is the class of transformations that is capable of aligning the images. The similarity metric determines the relative merit of the match. Then the search continues based on this result until a transformation is found whose similarity measure is satisfactory. The optimization process varies the parameters of the transformation model to maximize the matching criterion.

Figure I.1 shows the three basic elements of the registration methods: features, search strategies, and similarity measures. It has been known that there is no single best image registration technique for all types of data. Each technique has its own advantages and disadvantages for particular input data.

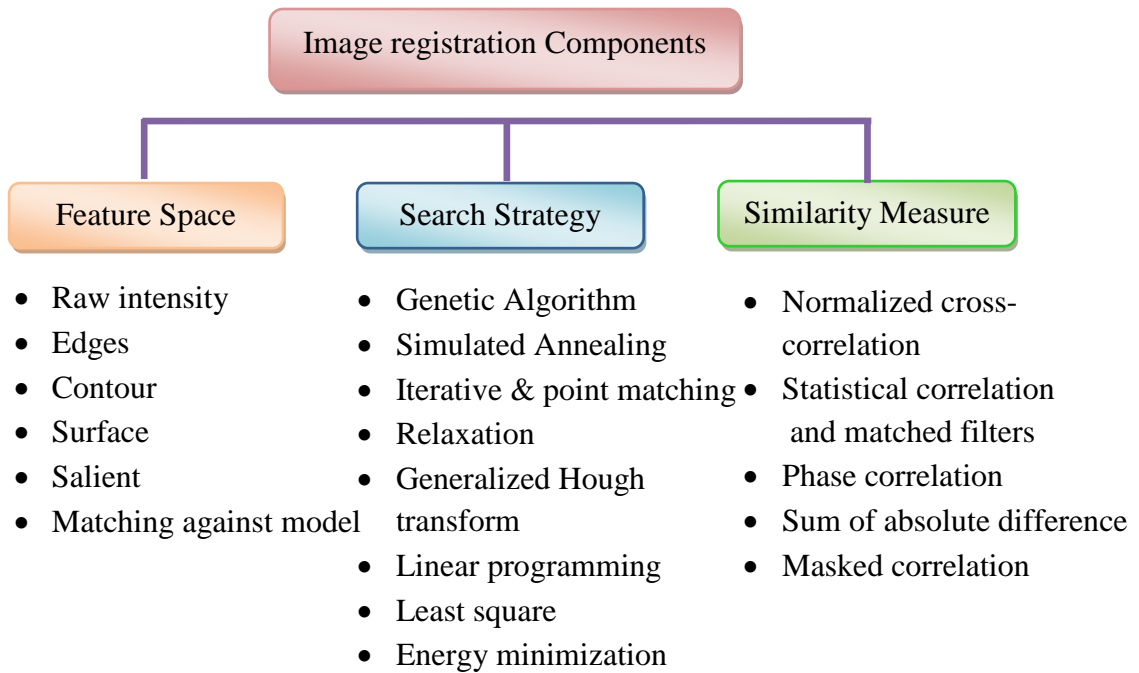


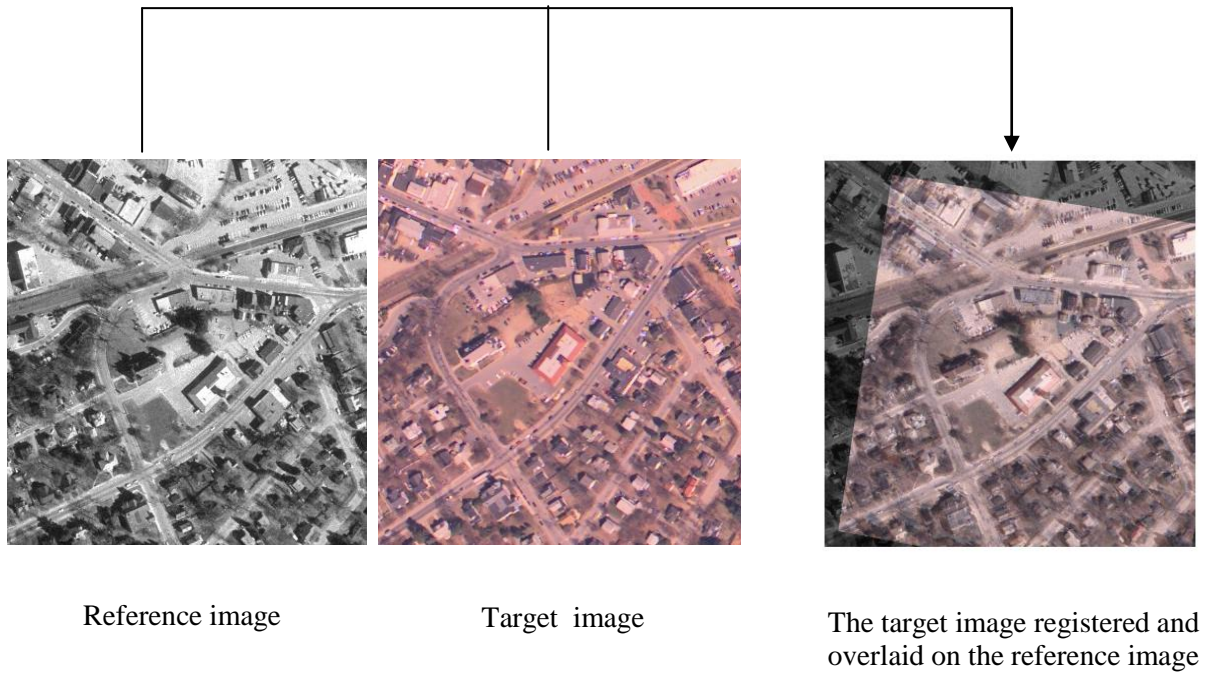
Figure I.1. Elements of image registration.

### I.3 Image Registration Concept

Image registration is the act of spatially mapping the coordinate system of one image to the coordinate system of another image. The one which is registered is called the reference image and the one which is to be matched to the reference image is called the sensed or input image. Figure I.2 shows the image registration concept.

Consider the problem of aligning the given two images  $F_R(x,y)$  and  $F_I(x,y)$  that we denote the reference and input images respectively with coordinates  $(x, y) \in R^2$ . To register the images is to find a geometric transformation  $T_p(.)$  of a certain class such that for all  $(x, y)$ ,  $F_R(T_p(x, y))$  best matches  $F_I(x, y)$  where  $P$  is a set of transformation parameters.





**Figure I.2.** Image Registration Concept.

Thus the registration problem is to find the optimal spatial transformation that matches the images, either for the purpose of determining the parameters of matching transformation or to expose the differences between the images. Depending on the application, spatial transformation mapping functions may take on many different forms. The most common general transformations are rigid, affine, projective, perspective, and global polynomial.

The most common global geometric distortions are the rigid geometrical transformations. These transformations preserve all distances and also preserve the straightness of lines. In addition, the overall geometric relationships between points do not change and, in consequence, the shapes of objects in the image do not change.

A rigid-body transformation is composed of a combination of a rotation, translation, and scale change that can be written as

$$T_P(x, y) = s \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \quad (I.1)$$

where  $(x_2, y_2)$  is the new transformed coordinate of  $(x_1, y_1)$ ,  $t_x$  and  $t_y$  are x-axis and y-axis translations, and  $s$  is a scale factor. Figure I.3 shows this transformation.

Affine transformations are more general than rigid-body and can tolerate more complex distortions (*Brown, 1992*). Rigid transformations account for object or sensor movement in which objects in the images maintain their relative shape and size. Translation and rotation transforms are usually caused by the different orientation of the sensor, while scaling transform is the effect of change in altitude of the sensor. The sensor distortion or the viewing angle may cause stretching and shearing (*Chalermwat, 1999*).

An affine transformation is a linear coordinate transformation that consists of the elementary operations which include translation, scaling and shear motion parameters. The general 2D affine transformation can be expressed as shown in the following equation where  $(x_2, y_2)$  is the new transformed coordinate of  $(x_1, y_1)$ .

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad (I.2)$$

Projective transformation is another type of transformation that is obtained when adding two more parameters to the above transformation and that introduces an additional distortion in the image (*Yasein, 2002*). This transformation describes what happens when viewing an object from some arbitrary viewpoint at a finite distance. Examples of affine and projective transformations are shown in the figure I.4.

Further, figure I.5 shows a cameraman image and the results of applying different geometric operations.

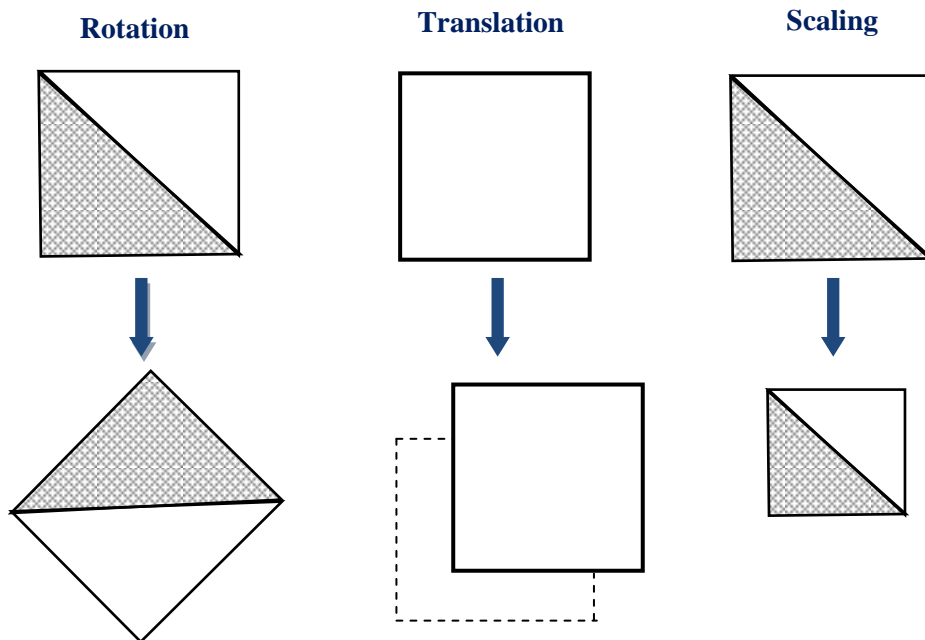


Figure I.3. Examples of rigid transformations.

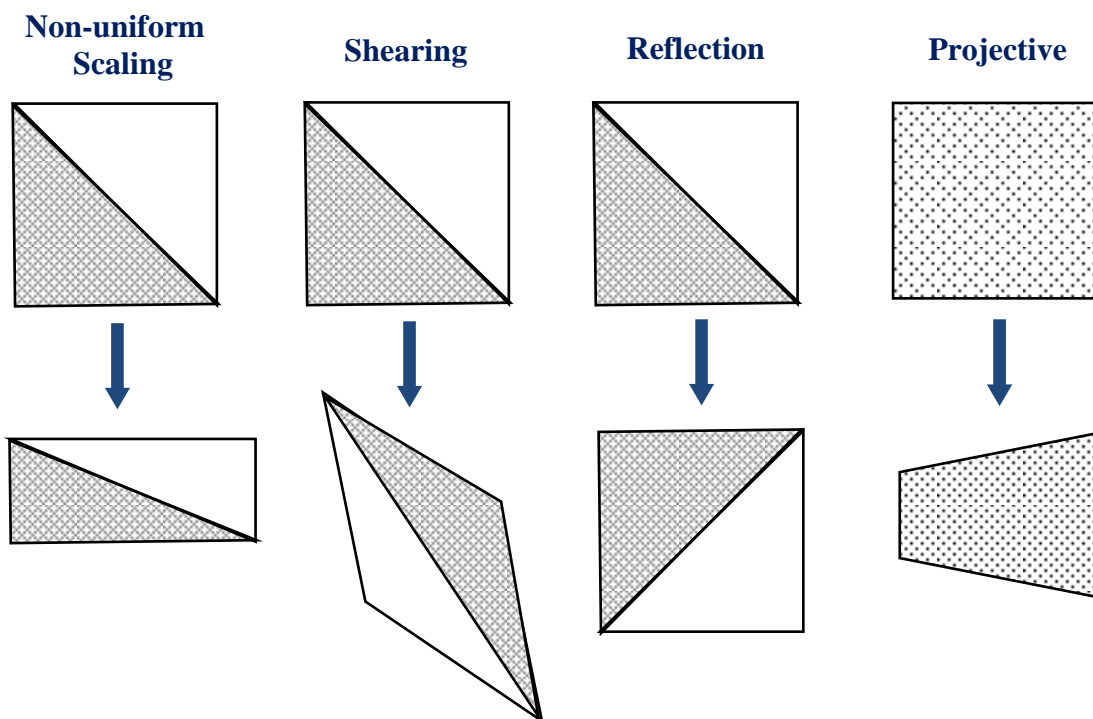


Figure I.4. Examples of affine and projective transformations.



**Figure I.5.** An image and the results of applying different geometric operations: (a) Original image, (b) Scaling down by 75%, (c) cropping, (d) rotation by  $10^\circ$ , (e) Translation, and (f) affine transformation.

## **I.4. Registration Algorithms Classification**

Many image registration techniques have been proposed in the literature. In general, existing image registration techniques can be categorized into two classes: area-based and feature-based methods. An extensive survey over image registration techniques can be found in (*Brown, 1992*), and (*Zitova & Flusser, 2003*).

Registration methods can be also categorized with respect to various criteria. The ones usually used are the application area, dimensionality of data, type and complexity of assumed image deformations, computational cost, and the essential ideas of the registration algorithm (*Zitova & Flusser 2003*). Here, the classification according to the essential ideas is chosen, considering the decomposition of the registration into the described following steps.

### ***I.4.1 Area based Methods***

These methods deal with the images without attempting to detect salient objects. In the area-based matching algorithms, a small window of pixels in the sensed image is compared statistically with windows of the same size as the reference image (*Althof & Wind 1997*). Usually the normalized cross-correlation or least-squares technique is used to measure the degree of match (*Pratt, 1974*). The centers of the matched windows are treated as control points, which can be used to solve for mapping function parameters between the reference and sensed images (*Li and al., 1995*).

Classical area-based methods like cross-correlation exploit for matching directly image intensities, without any structural analysis. Consequently, they are sensitive to the intensity changes, introduced for instance by noise, varying illumination, and/or by using different sensor types. From the geometric point of view, only shifts and small rotations between the images are allowed when using area based methods (although the area-based methods can be generalized to full rotations and scaling, it is practically meaningless because of an extreme computational load). To speed up the searching, area-based methods often employ pyramidal image representations and sophisticated optimization algorithms to find the maximum of the similarity matrix (*Zitova & Flusser, 2003*).

The area based method is simple and has a high precision. The prerequisite of the area based method is that gray level distribution of the sensed image and reference image must be similar because it is sensitive to geometric distortion and radiometric noise. Since gray values contain little explicit information about the object space, as a consequence, area-based matching methods are not reliable enough. Therefore, area-based methods are not well adapted to the problem of multisensor image registration since the gray-level characteristics of images to be matched can vary from sensor to sensor.

### ***1.4.2 Feature based Methods***

On the other hand, feature-based matching techniques do not use the gray values to describe matching entities, but use image features derived by a feature extraction algorithm. These features include edges, curves (*Can and al., 2002*), surfaces (*Thompson & Toga, 1996*), other salient features such as corners, line intersections, and points of high curvature, statistical features such as moment invariants or centroids, and higher level structural and syntactic descriptions.

There are two critical procedures generally involved in the feature-based techniques: feature extraction and feature correspondence. The basic building block of a feature based image registration scheme involves matching feature points that are extracted from a sensed image to their counter parts in a reference image. Features may be control points, corners, junctions or interest points. These features are also known as “visually salient points”. Feature based matching techniques not use the grey values to describe matching entities, but use image features derived by a feature extraction algorithm.

Compared with area based method, feature based method is more robust and reliable for the following reasons (*Schenk, 1999*): first, features are derived properties of the original gray-level images and are inherently unique; second, similarity is based on the attributes and/or relations, and is thus more invariant to illumination, reflectance, and geometry; third, features are sufficient for describing the image content. However, feature based method often requires sophisticated image processing for feature extraction and depends on the robustness of feature detection for reliable matching. A review of automatic image registration techniques can be also found in (*LeMoigne, 1995*).

Feature-based matching methods are typically applied when the local structural information is more significant than the information carried by the image intensities. They allow to registering images of completely different nature (like aerial photos and maps) and can handle complex between-image distortions. The common drawback of the feature-based methods is that the respective features might be hard to detect and/or unstable in time. The crucial point of all feature-based matching methods is to have discriminative and robust feature descriptors that are invariant to all assumed differences between the images (*Zitova & Flusser, 2003*).

Recently, registration methods using simultaneously both area-based and feature-based approaches have started to appear (*Hellier & Barillot, 2003*). A different approach is presented by Hellier, where optimal registration is achieved by finding the transformation that simultaneously matches image intensity values and relevant segmented brain structures.

### ***1.4.3. Transformation Model***

Image registration algorithms can also be classified according to the transformation model used to relate the reference image space with the target image space. The first broad category of transformation models includes linear transformations, which are a combination of translation, rotation, global scaling, shear and perspective components. Linear transformations are global in nature, thus not being able to model local deformations. Usually, perspective components are not needed for registration, so that in this case the linear transformation is an affine one. The second category includes 'elastic' or 'nonrigid' transformations. These transformations allow local warping of image features, thus providing support for local deformations. Nonrigid transformation approaches include polynomial wrapping, interpolation of smooth basis functions (thin-plate splines and wavelets), and physical continuum models.

### ***1.4.4 Single-modality vs Multi-modality***

Another useful classification is between single-modality and multi-modality registration algorithms. Single-modality registration algorithms are those intended to register images of the same modality (i.e. acquired using the same kind of imaging device), while multi-modality registration algorithms are those intended



to register images acquired using different imaging devices. There are several examples of multi-modality registration algorithms in the medical imaging field. Examples include registration of brain CT/MRI images or whole body PET/CT images for tumor localization, registration of contrast-enhanced CT images against non-contrast-enhanced CT images for segmentation of specific parts of the anatomy and registration of ultrasound and CT images for prostate localization in radiotherapy.

#### ***1.4.5. Frequency-domain Methods***

Other algorithms use the properties of the frequency-domain to directly determine shifts between two images. Applying the phase correlation method to a pair of overlapping images produces a third image which contains a single peak. The location of this peak corresponds to the relative translation between the two images. Unlike many spatial-domain algorithms, the phase correlation method is resilient to noise, occlusions, and other defects typical of medical or satellite images. Additionally, the phase correlation uses the fast Fourier transform to compute the cross-correlation between the two images, generally resulting in large performance gains. The method can be extended to determine affine rotation and scaling between two images by first converting the images to log-polar coordinates. Due to properties of the Fourier transform, the rotation and scaling parameters can be determined in a manner invariant to translation. This single feature makes phase-correlation methods highly attractive versus typical spatial methods, which must determine rotation, scaling, and translation simultaneously, often at the cost of reduced precision in all three.

### **1.5 Similarity Measures based Intensity Methods**

One of the most important components of any image registration method is the similarity metric. It is the criterion used to evaluate the similarity between two images and therefore is used to evaluate a given transformation. It is tightly related to the feature space. This is considered as a function  $F$  that measures the goodness of a given registration solution, that is, of a registration transformation  $f$ . The final performance of any image registration method will depend on its accurate estimation.



The choice of an image similarity measure depends on the nature of the images to be registered. Common examples of image similarity measures include Cross-correlation, Mutual information, Mean-square difference and Ratio Image Uniformity. Mutual information and its variant, Normalized Mutual Information, are the most popular image similarity measures for registration of multimodality images. Cross-correlation, Mean-square difference and Ratio Image Uniformity are commonly used for registration of images of the same modality.

### ***1.5.1 Cross-correlation***

Cross-correlation is one of the most common similarity metrics used in registration. It measures similarity by computing global statistics such as mean and variance, and it performs well if the two images are similar in nature, with an underlying linear relationship between the images intensities. The normalized cross-correlation function can be written as

$$C(A, B) = \frac{\sum_{i=0}^{N-1} \sum_{j=0}^{N-1} ((A_{ij} - \bar{A}) \times (B_{ij} - \bar{B}))}{\sqrt{\sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (A_{ij} - \bar{A})^2 \times \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (B_{ij} - \bar{B})^2}} \quad (1.3)$$

Where A and B are images of size N x N, while  $A_{ij}$  and  $B_{ij}$  denote pixel values in A and B respectively.

### ***1.5.2 Mutual Information***

Mutual information is a similarity metric originating from information theory. It has been extensively used in image registration, especially for multi-modal image registration tasks. On another hand, mutual information measures redundancy between two images by looking at their intensity distributions and it represents a measure of the relative entropy between two sets. A distribution with only a few large probabilities has a low entropy value; the maximum entropy value over a finite interval is achieved by a uniform distribution over that interval (*Pluim and al., 2000*).

Given two images A and B, the mutual information  $M(A,B)$  of the images is defined by

$$M(A,B) = H(A) + H(B) - H(A,B) \quad (I.4)$$

With  $H(A)$  and  $H(B)$  the marginal entropies of the separate images and  $H(A,B)$  the entropy of the joint image. We can now turn the problem of registering images A and B as maximising  $M(A,B)$  which means that the joint entropy  $H(A,B)$  is minimized.

Mutual information satisfies some important properties which confirm that the mutual information measures the degree of interdependence between two variables, reaching the lower bound when the two images are completely independent, and the upper bound when the images are the same. The strength of the mutual information similarity measure lies in the fact that no assumptions are made regarding the nature of the relationship between the image intensities in both modalities, except that such a relationship exists. This is not the case for correlation methods, which depend on a linear relationship between image intensities. For image registration, the assumption is that maximization of the Mutual information is equivalent to correctly registering the images. It is clear in equation I.4 that if the joint entropy of A and B are not affected by the transformation parameters, maximizing the Mutual information is equivalent to minimizing the joint entropy. This occurs when the images are correctly aligned. However, new combinations of intensity values from A and B will cause dispersion in the distribution. This dispersion leads to a higher joint entropy value, which in turn decreases the mutual information.

### ***1.5.3 Sum of Squared Differences***

When the images to be registered are from the same type, the image intensity at corresponding points between the two images should be similar. One of the simplest similarity measures is the sum of squared intensity differences (SSD) between images which is minimized during registration. Mathematically, this is defined by

$$SSD = \sum_{i,j} (A(i,j) - B(i,j))^2 \quad (I.5)$$

Where A is the fixed image intensity function, B represents the transformed image under the current transformation on consideration.

The optimal value of this measure of similarity is zero. Poor matches between images A and B result in large values. Such measure of similarity relies on the assumption that intensity representing the same homologous point must be the same in both images (*Ulysses & Conci, 2010*).

#### ***1.5.4 Sum of Absolute Differences***

The sum of Absolute Differences (SAD) is one of the simplest similarity measures. The sum of absolute differences works by taking the absolute value of the difference between each pixel in the original image A and the corresponding pixel in the transformed image under comparison. Smaller values of SAD represent more similar images.

$$SAD = \sum_{i,j} |A(i,j) - B(i,j)| \quad (1.6)$$

The SSD is very sensitive to a small number of pixels presenting very large intensity differences between images A and B. This could arise, for example, when contrast fluids are injected into the patient between the acquisitions of images A to B, or if the images are acquired during an intervention and instruments are in different positions relative to the subject in the two acquisitions. To reduce the impact generated by this sensitivity, the sum of absolute difference can be used (*Hajnal and al., 2001*).

#### **1.6 Conclusion**

Image registration is widely used in remote sensing, medical imaging, computer vision, video processing, and many others. To register two images, a transformation must be found so that each point in one image can be mapped to a point in the second. This mapping must “optimally” align the two images where optimally depends on what needs to be matched.

Misalignment can occur due to any of the following reasons. First, images may be taken at the same time but acquired from several sensors, each having different distortion properties. Second, images may be taken from one sensor at different times and at various viewing geometries. Furthermore, sensor motion will give rise to distortion as well. Geometric transformations were originally introduced to correct these distortions and to allow the accurate determination of spatial relationships and scale.

Different search strategies can be applied as well, in order to improve the results and decrease the costs of the registration method. At the beginning of the search procedure, the transformation parameters are searched for in a coarse manner using simple search strategy. The results are then refined by an accurate, though more complex, search strategy. Recent works have investigated the global search technique as a genetic algorithm to find the best transformation parameters. The power of GAs lies in their robustness. The topic of the next chapter is to study this technique.

# CHAPTER II

# GENETIC ALGORITHMS

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## II.1 Introduction

The world as we see it today, with its variety of different creatures, its individuals highly adapted to their environment, with its ecological balance (under the optimistic assumption that there is still one), is the product of a three billion years experiment we call evolution, a process based on sexual and asexual reproduction, natural selection, mutation, and so on. If we look inside, the complexity and adaptability of today's creatures has been achieved by refining and combining the genetic material.

Over a long period of time, the theory of natural selection first down by Charles Darwin (*Darwin, 1859*) explains the process of evolution and adaptation based on "the survival of the fittest" phenomenon. Individuals with characteristics that were suitable for survival of their environment had higher life expectancy and thus reproduce more. This led to a proliferation of their suitable characteristics in the subsequent generations. Individuals became "fitter" for survival as generation progressed due to the process of combination of the characteristics of parents during reproduction. On the other hand, those individuals without such characteristics did not survive long and produced less.

*"I have called this principle, by which each slight variation, if useful, is preserved, by the term Natural Selection."*

Charles Robert Darwin, the Origin of Species, 1859

Genetic Algorithms (GAs) are adaptive heuristic search algorithms based on the evolutionary ideas of natural selection and genetics. They represent an intelligent exploitation of a random search used to solve optimization problems. Although randomized, GAs are by no means random, instead they exploit historical information to direct the search into the region of better performance within the search space.

Generally speaking, GAs are simulations of evolution, of what kind ever. In most cases, however, genetic algorithms are nothing else than probabilistic optimization methods which are based on the principles of evolution. This idea appears first in 1967 in J. D. Bagley's thesis "The Behavior of Adaptive Systems Which Employ Genetic and Correlative Algorithms" (*Bagley, 1967*).

The theory and applicability was then strongly influenced by J. H. Holland, who can be considered as the pioneer of GAs (*Holland, 1975*). Since then, this field has witnessed a tremendous development. The purpose of this chapter is to give a comprehensive overview of this class of methods, the main fundamentals and basic principles of GAs, and their applications in optimization.

### II.2 Optimization and Evolutionary Algorithms

The buzzword doing the beats at all hierarchical levels of the industry today is optimization. Calculus had been the reigning emperor of optimization techniques, until recently. One such optimization technique which mimics the natural process of evolution is Evolutionary Computing. Evolutionary Computation represents powerful search and optimization paradigm influenced by biological mechanisms of evolution. Evolutionary algorithms include a range of nature inspired techniques that attempt to simulate the way in which some processes occur in nature. Some of the natural processes in which evolutionary algorithms are based are for example: the evolution of species, the organisation of insect colonies, the organisation and evolution of cultural systems, the working of immune systems and many others. Then, evolutionary algorithms are techniques that manage a population of individuals (for example, each individual may represent one solution to a problem) and by means of reproduction, selection, heritability, feedback and other principles attempt to generate better individuals as the population evolves.

Evolutionary algorithms have been applied to a wide range of problems including optimisation, search and design. Among the feature characteristics of these techniques, there is their capability to explore large search spaces thanks to the fact that they are stochastic population-based methods which makes them suitable for approaching complex problems such as optimisation and design.

GAs represent the main paradigm of evolutionary algorithms. Prof. John Holland of the University of Michigan viewed the process of Biological Evolution as a process of optimization, where nature selects the best genetic settings to survive in the next generation of offsprings. These offsprings are then optimized further to give successively better offsprings.

A GA similarly selects the most optimal solutions from a set, and uses the genetic operators of Crossover & Mutation to generate further solutions. Each such solution is "more" optimal than its predecessors. An important feature of biological evolution is robustness - which is what genetic algorithms strive to achieve.

*"Computer programs that evolve in ways that resemble natural selection can solve complex problems even their creators do not fully understand"*

John H. Holland, 1975

The performance of GA, like any global optimization algorithm, depends on the mechanism for balancing the two conflicting objectives, which are exploiting the best solutions found so far and at the same time exploring the search space for promising solutions. The power of GA comes from their ability to combine both exploration and exploitation in an optimal way.

GAs distinguish themselves from other optimizing tools because of their implicit parallelism, diversity and intensification. Parallelism and diversity are achieved by using a population of solutions instead of a single solution, which makes the GA one of the best global optimization tools. Intensification consists in preserving good solutions and combining their good features to produce better solutions through selection and crossover. This desirable feature makes the GA a more efficient searching tool compared to other merely exploratory and costlier global optimization methods which are based on exhaustive-like and random mutation search.

Compared with traditional continuous optimization methods, such as Newton or gradient descent methods, we can state the following significant differences:

1. GAs manipulate coded versions of the problem parameters instead of the parameters themselves.
2. While almost all conventional methods search from a single point, GAs always operate on a whole population of points (strings). This contributes much to the robustness of genetic algorithms. It improves the chance of



reaching the global optimum and, vice versa, reduces the risk of becoming trapped in a local stationary point.

3. GAs do not use any auxiliary information about the objective function value such as derivatives. Therefore, they can be applied to any kind of continuous or discrete optimization problem. The only thing to be done is to specify a meaningful decoding function.
4. GAs use probabilistic transition operators while conventional methods for continuous optimization apply deterministic transition operators. More specifically, the way a new generation is computed from the actual one has some random components.

### II.3 Mathematical Formulation

Applying mathematics to a problem of the real world mostly means, at first, modeling the problem mathematically, may be with hard restrictions, idealizations, or simplifications, then solving the mathematical problem, and finally drawing conclusions about the real problem based on the solutions of the mathematical problem.

As a first approach, let us restrict to the view that genetic algorithms are optimization methods. In general, optimization problems are given in the following form:

Find an  $x_0 \in X$  such that  $f$  is maximal in  $x_0$ , where  $f : X \rightarrow \mathbb{R}$  is an arbitrary real-valued function, i.e.

$$f(x_0) = \max_{x \in X} f(x) \tag{II.1}$$

In practice, it is sometimes almost impossible to obtain global solutions in the strict sense of equation (II.1). Depending on the actual problem, it can be sufficient to have a local maximum or to be at least close to a local or global maximum.

So, let us assume in the following that we are interested in values  $x$  where the objective function  $f$  is “as high as possible”. The search space  $X$  can be seen in

direct analogy to the set of competing individuals in the real world, where  $f$  is the function which assigns a value of “fitness” to each individual (this is, of course, a serious simplification).

In the real world, reproduction and adaptation is carried out on the level of genetic information. Consequently, GAs do not operate on the values in the search space  $X$ , but on some coded versions of them (strings for simplicity).

Assume  $S$  to be a set of strings. Let  $X$  be the search space of an optimization problem as above, then a function

$$\begin{aligned} C: X &\longrightarrow S \\ x &\longrightarrow C(x) \end{aligned}$$

is called coding function.

Conversely, a function

$$\begin{aligned} \tilde{C}: S &\longrightarrow X \\ s &\longrightarrow \tilde{C}(s) \end{aligned}$$

is called decoding function.

In practice, coding and decoding functions, which have to be specified depending on the needs of the actual problem, are not necessarily bijective. However, it is in most of the cases useful to work with injective decoding functions

## **II.4 Definition & Terminology**

### ***II.4.1 GAs Definition***

GA is a global search technique that imitates natural biological evolution. The algorithm, starting from an initial population of potential solutions and preserving the best individuals, produces new population obtaining better and better approximations to a solution. The individuals are represented by their genetic material or genotype. This is accomplished by choosing a suitable representation scheme to code the possible solutions into individuals. Commonly, a solution is

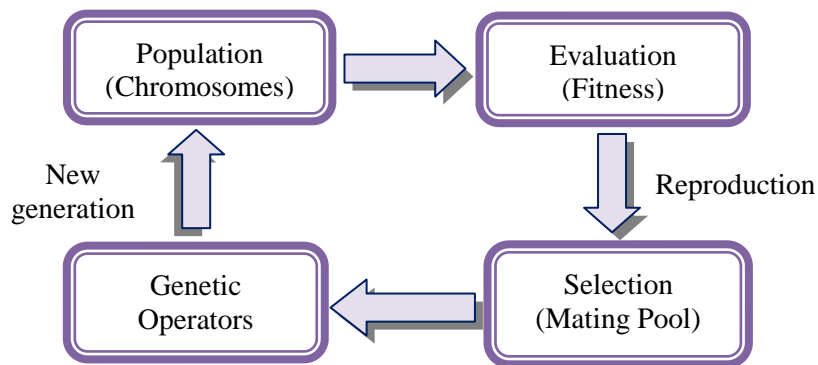
coded into a genotype consisting of a binary chromosome string with a predetermined number of bits which determines the problem's size.

The fitness of chromosome is the way that is linked to the predefined problem or objective function. Hence, each individual is evaluated using a fitness value which is a measure of how well adapted this individual is to its environment. According to the evolutionary theories, the chromosomes which only have a good fitness are likely to survive and to generate the offsprings and pass strength to them by the genetic operators. The higher the level of fitness of an individual, the greater its chance of being selected. This fitness driven selection leads to the evolution of populations of individuals that are better than the population of their ancestors.

At each generation, individuals are selected and bred together, resulting in a new set of approximations. The modification of the population is done using an iterative application of three genetic operators that provide general exploratory heuristics which evolve the population from one generation to the next. The genetic algorithm then creates a population of solutions and applies genetic operators such as mutation and crossover to evolve the solutions in order to find the best one(s). These operators are selection, crossover, and mutation. This continues until a suitable solution has been found or a certain number of generations have passed, depending on the needs of the programmer. It is clear that GAs can be used to perform the optimization process in the problems for finding the best solution from a set that contains finite possible solutions.

Figure II.1 shows the GA scheme. Furthermore, the simple GA algorithm can be expressed as follows:

1. Randomly initialize population  $p(t)$ .
2. Determine fitness of population  $p(t)$ .
3. Repeat
  1. Select parents from population  $p(t)$
  2. Perform crossover on parents creating population  $p(t+1)$
  3. Perform mutation of population  $p(t+1)$
  4. Determine fitness of population  $p(t+1)$
4. Until best individual is good enough.



**Figure II.1:** A GA scheme.

### II.4.2 GAs Terminology

A GA borrows its name from the natural genetic system. A GA uses a direct analogy of natural behavior. The GA pool contains a number of individuals called chromosomes. Each chromosome encoded from the parameters holds the potential solution to a particular problem. In nature, individuals are determined by their *genes* in their *chromosomes*. In computing, genes and chromosomes can be represented by strings. The most commonly used alphabet of the strings is binary, but other alphabets are also used, e.g, integer or real valued numbers, depending on which is the most suitable for a given problem. The following table gives a list of different expressions, which are common in genetics, along with their equivalent in the framework of GAs:

Natural Evolution	Genetic Algorithms
genotype	coded string
phenotype	decoded string
chromosome	String, creature
gene	string position
allele	value at a certain position
fitness	objective function value

**Table II. 1.** GA terminology.

## II.5 Basics of Genetic Algorithms

The GA generally includes three fundamental genetic operations, which are selection, crossover and mutation. The purpose of those operations is to modify the chosen solutions and select the most appropriate offspring to pass on the succeeding generation until no better fitted solutions are possible. The purpose of this section is to describe these operators.

### II.5.1 Selection

According to the natural selection theory, individuals that are better fit to a given environment are more likely to survive. In this way, the selection directs the genetic search toward promising regions in the search space and that will improve the performance of GAs. Many selection methods have been proposed, examined and compared. The most common types are: roulette wheel selection, tournament selection, rank selection and steady state selection.

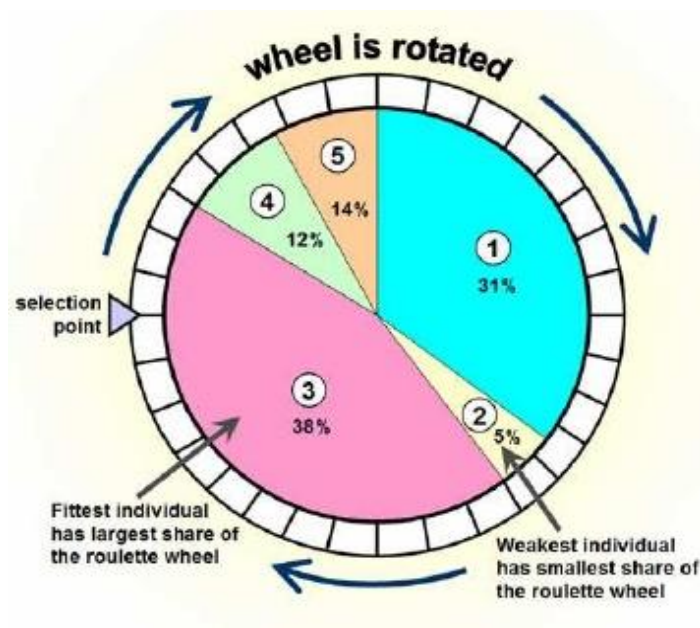
The roulette wheel selection is the most popular, and known as the fitness proportionate selection. The idea behind the roulette wheel selection technique is that each individual is given a chance to become a parent in proportion according to a probability that is proportional to its fitness. The chromosomes with higher fitness values will be selected more times and have more chances to reproduce in the next generations. The probability of selection of an individual is given in equation (II.3), where  $f(\text{parent}_i)$  is the fitness of the  $i$ th parent:

$$P_{\text{selection}} = \frac{f(\text{parent}_i)}{\sum_i f(\text{parent}_i)} \quad (\text{II.2})$$

The probability of selecting an individual from the population is purely a function of the relative fitness of the individual. Individuals with high fitness will participate in the production of the next generation more often than less fit individuals.

It is called the roulette wheel selection as the chances of selecting a parent can be seen as spinning a roulette wheel with the size of the slot for each parent being

proportional to its fitness. Each member of the population is allocated the amount of space on the wheel that reflects its relative fitness. The fitter the individual, the greater the area on the roulette wheel that is occupied by the individual. Obviously those with the largest fitness (slot sizes) have more chances of being chosen. Thus, it is possible for one member to dominate all the others and get selected a high proportion of the time. Figure II.2 gives a graphical hint how this roulette wheel game works where the number of alternatives  $m$  is 5. The numbers inside the arcs correspond to the probabilities to which the alternative is selected.



**Figure II.2.** A graphical representation of roulette wheel selection.

The roulette wheel selection scheme can be implemented as follows:

1. Evaluate the fitness,  $f_i$ , of each individual in the population.
2. Compute the probability (slot size),  $p_i$ , of selecting each member of the population:  $p_i = f_i / \sum_{j=1}^n f_j$ , where  $n$  is the population size.
3. Calculate the cumulative probability,  $q_i$ , for each individual:  $q_i = \sum_{j=1}^i p_j$ .
4. Generate a uniform random number,  $r \in [0, 1]$ .
5. If  $r < q_1$  then select the first chromosome,  $x_1$ , else select the individual  $x_i$  such that  $q_{i-1} < r \leq q_i$ .
6. Repeat steps 4–5  $n$  times to create  $n$  candidates in the mating pool.

### II.5.2 Crossover

In sexual reproduction, as it appears in the real world, the genetic material of the two parents is mixed when the gametes of the parents merge. Usually, chromosomes are randomly split and merged, with the consequence that some genes of a child come from one parent while others come from the other parents.

Crossover operation is used to generate the next generation of candidate solutions. In this operation, it involves the swapping of the genetic material between the two parent strings. It is a very powerful tool for introducing new genetic material and maintaining genetic diversity, but with the outstanding property that good parents also produce well-performing children or even better ones. Several investigations have come to the conclusion that crossover is the reason why sexually reproducing species have adapted faster than asexually reproducing ones. In most GAs, individuals are represented by fixed-length strings and crossover operates on pairs of individuals (parents) to produce new strings (offspring) by exchanging segments from the parents' strings. A commonly used method for crossover is called single point crossover. In this method, a bit position along the two strings is selected randomly and the two parent strings exchange their genetic materials as illustrated in figure II.3.

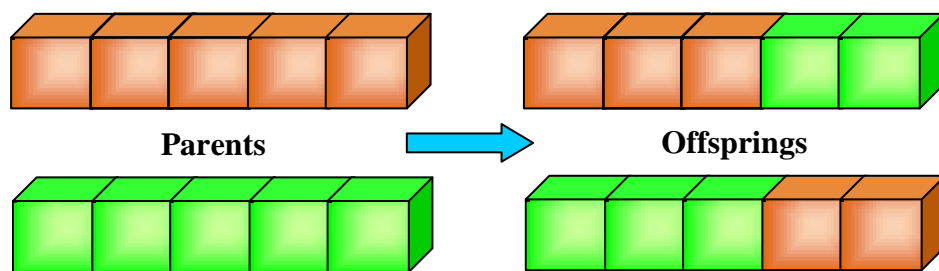


Figure II.3. One point crossover.

One-point crossover is a simple and often-used method for GAs which operate on binary strings. For other problems or different codings, other crossover methods can be useful or even necessary. We mention just a small collection of them, for more details see (*Geyer, 1995*), (*Goldberg, 1989*):

***N-point crossover:*** Instead of only one, N breaking points are chosen randomly. Every second section is swapped. Among this class, a two-point crossover is particularly important

***Segmented crossover:*** Similar to the N-point crossover with the difference that the number of breaking points can vary.

***Uniform crossover:*** For each position, it is decided randomly if the positions are swapped.

***Shuffle crossover:*** First, a randomly chosen permutation is applied to the two parents, then an N-point crossover is applied to the shuffled parents, finally, the shuffled children are transformed back with the inverse permutation.

### ***II.5.3 Mutation***

After selection and crossover, you now have a new population full of individuals. Some are directly copied, and others are produced by crossover. In order to ensure that the individuals are not all exactly the same, you allow for a small chance of mutation. The mutation operation takes place with a certain probability, which, in accordance with its biological equivalent, typically occurs with a very low probability. It alters one or more bit values at randomly selected locations in randomly selected strings as shown in the figure II.4.

Mutation allows for the possibility that non-existing features from both parent strings may be created and passed to their children. Without an operator of this type, some possibly important regions of the search space may never be explored. So, mutation is a common operator used to help preserve diversity in the population by finding new points in the search space to evaluate.

Therefore, the mutation operator enhances the ability of GAs to find a near optimal solution in a given problem by maintaining a sufficient level of genetic variety in the searching for the best solution. It serves as an *insurance policy* and helps prevent the loss of genetic materials (*Goldberg, 1989*).



Similar to the case of crossover, the choice of the appropriate mutation technique depends on the coding and the problem itself. We mention a few alternatives, more details can be found in (Geyer, 1995) and (Goldberg, 1989) again:

**Inversion of single bits:** With probability  $P_m$ , one randomly chosen bit is negated.

**Bitwise inversion:** The whole string is inverted bit by bit with probability  $P_m$ .

**Random selection:** With probability  $P_m$ , the string is replaced by a randomly chosen one.

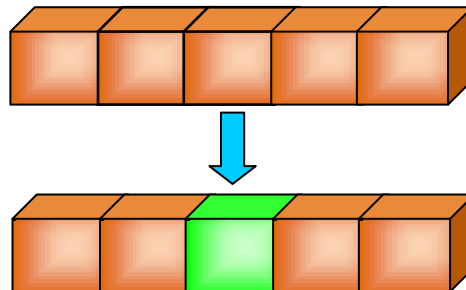


Figure II.4. Mutation operator.

### Effects of genetic operators

- Using selection alone will tend to fill the population with copies of the best individual from the population.
- Using selection and crossover operators will tend to cause the algorithms to converge on a good but sub-optimal solution.
- Using mutation alone induces a random walk through the search space.
- Using selection and mutation creates a parallel, noise tolerant, hill-climbing algorithm.

## **II.6 Methodology and Basic GA Parameters**

The GAs provide a directed random search in complex landscapes. There are two important issues with respect to search strategies: exploration (investigate new and unknown areas in search space) and exploitation (make use of knowledge of solutions previously found in search space to help in find better solutions). This can be done by making genetic operators perform essentially a blind search; with a hope that selection operators direct the genetic search toward the desirable area of solution space.

Basically, A GA requires two elements for a given problem:

- ◆ encoding of candidate structures (solutions)
- ◆ method of evaluating the relative performance of candidate structure, for identifying the better solutions

A GA codes parameters of the search space as binary strings of fixed length. It employs a population of strings initialized at random, which evolve to the next generation by genetic operators such as selection, crossover and mutation. The fitness function evaluates the quality of solutions coded by strings. Selection allows strings with higher fitness to appear with higher probability in the next generation. Crossover combines two parents by exchanging parts of their strings, starting from a randomly chosen crossover point. This leads to new solutions inheriting desirable qualities from both parents. Mutation flips single bits in a string, which prevents the GA from premature convergence, by exploiting new regions in the search space. The GA tends to take advantage of the fittest solutions by giving them greater weight, and concentrating the search in the regions which lead to fitter structures, and hence better solutions of the problem.

Finding good parameter settings that work for a particular problem is not a trivial task. The critical factors are to determine robust parameter settings for population size, encoding, selection criteria, genetic operator probabilities and evaluation (fitness) normalization techniques.

Generally, Basic parameters of GAs includes: population size, probability and type of crossover, and probability and types of mutation. By varying these parameters, the convergence of the problem may be altered.

Thus, to maintain the robustness of the algorithm, it is important to assign appropriate values for these parameters. Much attention has been focused on finding the theoretical relationship among these parameters. Schwefel in (*Schwefel, 1981*) developed theoretical models for optimal mutation rates with respect to convergence and convergence rates in the context of function optimization. De Jong presented theoretical and empirical results on the interacting roles of the population size and crossover in GA (*DeJong & Spears, 1990*).

Crossover rate should be generally about 65% - 95%, (Crossover probability says how often will be crossover performed). Mutation probability says how often will be parts of chromosome mutated. A variety of authors have attempted to determine fixed values for the mutation rate which will yield good results across a range of problems (*Grefenstette, 1986*). Mutation is applied with low probability, typically in the range 0.01 – 0.1. Particularly important parameters of GAs are the population size and the number of generations. Population size says how many chromosomes are in population (in one generation). If there are too few chromosomes, GAs have a few possibilities to perform crossover and only a small part of search space is explored and therefore increases the risk of converging prematurely to a local minima, since the population does not have enough genetic material to sufficiently cover the problem space. On the other hand, if there are too many chromosomes, the GA has a greater chance of finding the global optimum but slows down. Research shows that after some limit (which depends mainly on encoding and the problem) it is not useful to increase the population size, because it does not make solving the problem faster.

### II.7 Applications of Genetic Algorithms

GAs are adaptive methods which may be used to solve search and optimization problems. The power of GAs comes from the fact that the technique is robust and can deal successfully with a wide range of problem areas, including those which are difficult for other methods to solve. GAs in various forms are implemented in a wide range of problems including the following:

- ❖ Optimization: GAs have been used in a wide variety of optimization tasks, including numerical optimization and combinatorial optimization problems such as circuit design and job shop scheduling.
- ❖ Automatic Programming: GAs have been used to evolve computer programs for special tasks and to design other computational structures cellular automata and sorting networks.
- ❖ Machine and robot learning: GAs have been used for many machine learning applications, including classification and prediction tasks such as the prediction of dynamical systems, weather prediction , and prediction of protein structure. GAs have also been used to design neural networks, and to evolve rules for learning classifier systems or symbolic production systems and to design and control robots (*Grefenstette, 1995*).
- ❖ Economic models: GAs have been used to model processes of innovation, the development of bidding strategies, and the emergence of economic markets.
- ❖ Ecological models: GAs have been used to model ecological phenomena such as biological arms races, host-parasite co-evolution, symbiosis, and resource flow in ecologies.
- ❖ Models of social systems: GAs have been used to study evolutionary aspects of social systems, such as the evolution of cooperation, the evolution of communication, and trail-following behavior in ants.
- ❖ Scheduling: Facility, Production, Job, and Transportation Scheduling.
- ❖ Design: Circuit board layout, Communication Network design, keyboard layout, Parametric design in aircraft.
- ❖ Control: Missile evasion, Gas pipeline control, Pole balancing.
- ❖ Combinatorial Optimization: TSP, Bin Packing, Set Covering, Graph Bisection, Routing,
- ❖ Signal Processing as in Filter Design: (*Meskine & Taleb, 2004*), (*Meskine & Taleb, 2006*).
- ❖ Image Processing: Pattern recognition.
- ❖ Business: Economic Forecasting; Evaluating credit risks, Detecting stolen credit cards before customer reports it is stolen.
- ❖ Medical: Studying health risks for a population exposed to toxins.

## **II.8 Conclusion**

A GA is a search heuristic that mimics the process of natural evolution. This heuristic is routinely used to generate useful solutions to optimization and search problems. Genetic algorithms belong to the larger class of evolutionary algorithms, which generate solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover.

Genetic Algorithms have been widely studied, experimented and applied in many fields in engineering worlds. Not only does a GA provide an alternative method to solving problem, it consistently outperforms other traditional methods in most of the problems link. Many of the real world problems involve finding optimal parameters, which might prove difficult for traditional methods but ideal for GAs. As such, they represent an intelligent exploitation of a random search within a defined search space to solve a problem.

It is important to emphasis that the GA is non-deterministic: different runs yield different results depending on the seed of the random number generator. However, when the problem does not have one single solution, or when different solutions close to the best one are acceptable, the GA is useful and often works better than traditional methods.

In the broad area of global optimization methods, Genetic Algorithms form a widely accepted trade-off between global and local search strategy. They were chosen here and they are well-investigated and have proven their applicability in many fields, such as in this thesis where the application is in the field of image registration.

# CHAPTER III

## **MULTI- RESOLUTION TRANSFORMS**

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### III.1 Introduction

Wavelets are classified as a linear transform that is capable of displaying the transformed output at multiple resolutions depending on the point of time/space and at desired frequency.

In the 1D case, wavelets are used for signal processing by the virtue that wavelets can store more frequency information with less coefficients and reconstruction is only limited by the coefficients that are available. Wavelets can be naively extended to the 2D case by means of separable functions, but there is limited directional information stored in a regular 2D wavelet transform. Because of the separability limitations, only a horizontal, vertical, and 45 degree component can be easily determined. Incidentally, edges can be seen easily, but directional information about the edge is not known. Because of this, it takes more coefficients to do a proper reconstruction of the edges (*Antoine and al., 1993*).

Typically, a separable 2D wavelet transform provides:

- Multiresolution, which is the ability to visualize the transform with varying resolution from coarse to fine
- Localization, which is the ability of the basis elements to be localized in both the spatial and frequency domains
- Critical sampling, which is the ability of the basis elements to have little redundancy.

However, it is not capable of providing:

- Directionality, which is having basis elements defined in a variety of directions.
- Anisotropy, which is having basis elements defined in various aspect ratios and shapes.

In recent years, the fast growth of multiscale geometric analysis has brought out abundant tools for image processing, such as the contourlet transform. Indeed, the contourlet transform is a new efficient image decomposition scheme, which provides sparse representation at both spatial and directional resolutions. The contourlet transform employs Laplacian pyramids to achieve multiresolution decomposition and directional filter banks to yield directional decomposition, such that, the image is represented as a set of directional subbands at multiple

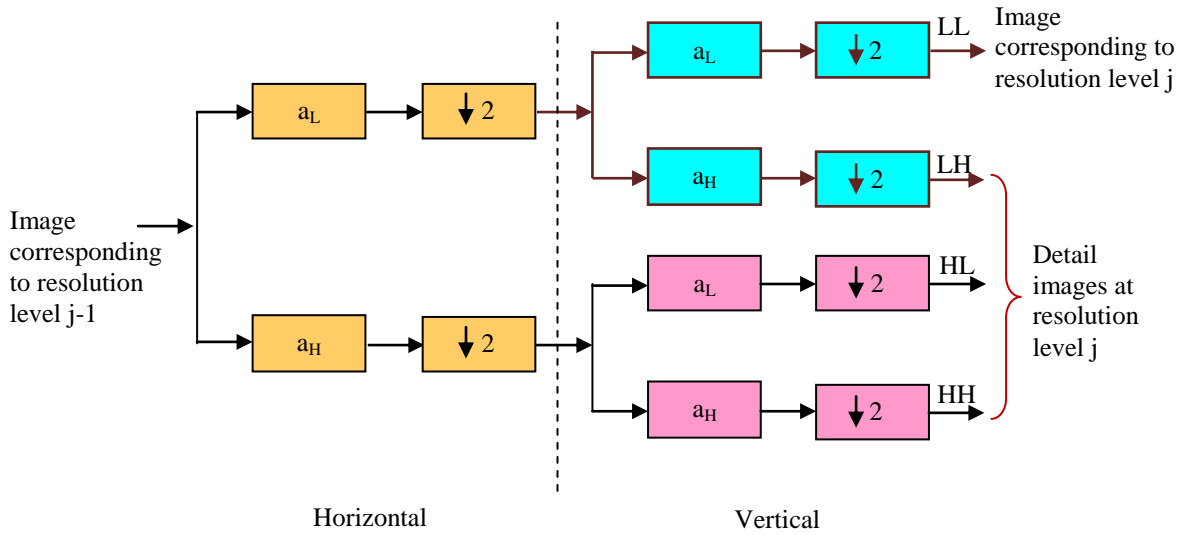
scales. The contourlet transform with its an extra feature of directionality achieves better results than the discrete wavelet transform and yields new potentials in image processing applications (*Do & Vetterli, 2005*). Another transform is introduced which is a shift-invariant version of the contourlet transform named nonsubsampling contourlet transform to overcome of the problem of sampling which causes translational invariance of the contourlet transform. Therefore, the main part of this chapter is to present a comprehensive study of these multiresolution transforms.

### **III.2 Multi-resolution Representation of the Image**

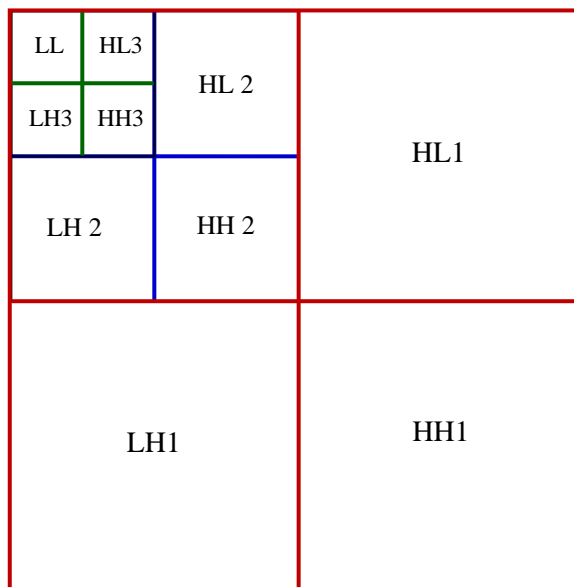
The multi-resolution scheme developed by Mallat (*Mallat, 1993*) provides a very fast algorithm which increases the importance of wavelets for on-line processing of imagery data. The wavelet based multi-resolution preserves most of the important features of the original data even at a low resolution. It also eliminates weak features in higher resolution while highlighting strong image features.

The wavelet transform is a powerful tool for multi-resolution analysis. The ordinary wavelet transform consists of filtering and downsampling operations. The necessary two-dimensional filtering operations are implemented via separable filters as shown in fig.III.1(a). The Low-Low (LL) sub-image is created by lowpass filtering. It is a coarse approximation of our original image: a direct result of the lowpass filtering is smoothing of the image edges in both directions. The Low-High (LH) subimage blurs the horizontal edges because the rows are lowpass filtered, but it preserves vertical ones because the columns are highpass filtered. The other sub-image known as High-Low (HL) sub-image preserves the horizontal edges but blurs the vertical ones. We use the LH and HL subimages to extract the vertical and horizontal edges. The sum of two subimages gives us information about the horizontal and vertical edges of the original image. The remaining High-High (HH) subimage preserves the noise in the original image and hence it is not employed. Figure III.1(b) presents the three level of decomposition using the wavelet transform. At each level of the wavelet decomposition, four new images are created from the original  $N \times N$ -pixel image. The size of these new images is reduced to  $\frac{1}{4}$  of the original size, i.e., the new size is  $N/2 \times N/2$ . Thus, the four images produced from each decomposition level are LL, LH, HL, and HH. In the wavelet decomposition, only the LL image is used to produce the next level of decomposition.





(a)



(b)

**Figure III.1.** Structure of wavelets decomposition: (a) Stage filter of two-dimensional wavelets, (b) Sub-bands of three level wavelet decomposition.

### III.3 Contourlet Transform

The standard two-dimensional wavelet transform is widely used in image processing. However, this technique fails to capture efficiently phenomena in images in directions other than the horizontal and vertical. Recently Do and Vetterli proposed an efficient directional multi-resolution image representation called the contourlet transform (*Do & Vetterli, 2005*). The contourlet transform is an extension of the Cartesian wavelet transform in two dimensions using multiscale and directional filter banks, which offers a multi-resolution and directional decomposition. The transform employs the Laplacian pyramids to achieve multi-resolution decomposition and directional filter banks to achieve directional decomposition.

#### III.3.1 Laplacian Pyramid

One way of achieving a multiscale decomposition is to use a Laplacian pyramid (LP), introduced by (*Burt & Adelson, 1983*). The LP decomposition at each level generates a down sampled lowpass version of the original and the difference between the original and the prediction, resulting in a bandpass image as shown in Fig. III.2. In this figure, ‘H’ and ‘G’ are called analysis and synthesis filters and ‘M’ is the sampling matrix. The process can be iterated on the coarse version.

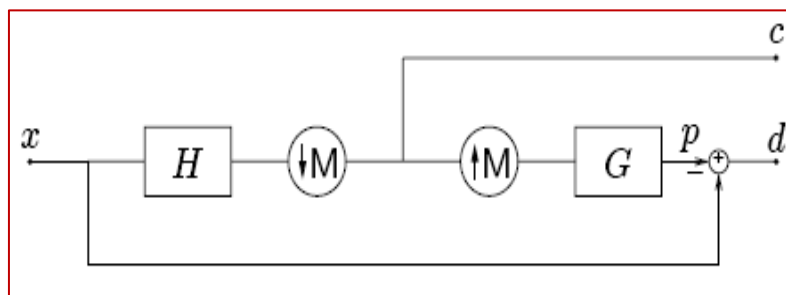
In figure III.2(a) the outputs are a coarse approximation ‘a’ and a difference ‘b’ between the original signal and the prediction. The process can be iterated by decomposing the coarse version repeatedly.

A drawback of the LP is the implicit oversampling. However, in contrast to the critically sampled wavelet scheme, the LP has the distinguishing feature that each pyramid level generates only one bandpass image (even for multidimensional cases), which does not have “scrambled” frequencies. This frequency scrambling happens in the wavelet filter bank when a highpass channel, after downsampling, is folded back into the low frequency band, and thus its spectrum is reflected. In the LP, this effect is avoided by downsampling the lowpass channel only.

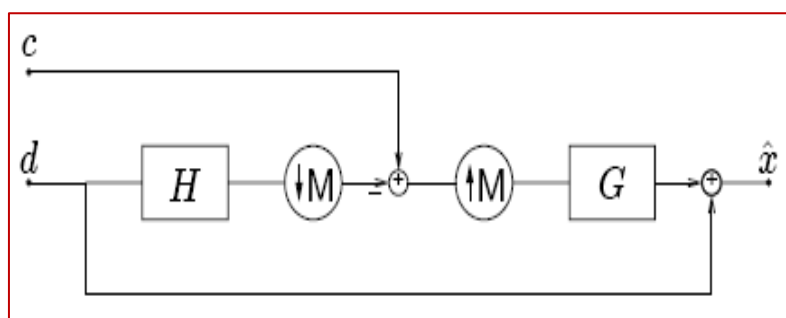
In the theory of frames and oversampled filter banks, it is showed that the LP with orthogonal filters (  $h[n] = g[-n]$  ) provides a tigh frame with frame bounds equal to 1. In this case, Do & Vitterli propose the use of the optimal linear reconstruction using the dual frame operator (or pseudo-inverse), which is symmetrical with the forward transform as shown in Fig. III.2(b).

The new reconstruction differs from the usual method, where the signal is obtained by simply adding back the difference to the prediction from the coarse signal, and was shown to achieve significant improvement over the usual reconstruction in the presence of noise.

Thus the Laplacian pyramid is a set of band pass filters. By repeating these steps several times a sequence of images, are obtained. If these images are stacked one above another, the result is a tapering pyramid data structure, as shown in Fig. III.3 and hence the name. The Laplacian pyramid can thus be used to represent images as a series of bandpass filtered images, each sampled at successively sparser densities. It is frequently used in image processing and pattern recognition tasks because of its ease of computation.

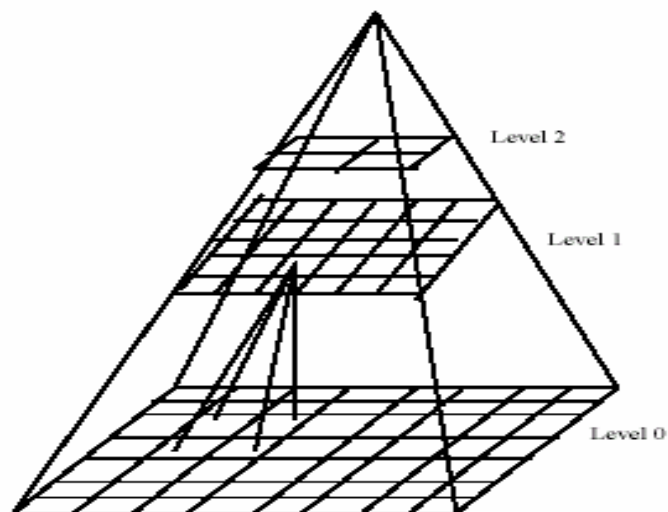


(a)



(b)

**Figure III.2.** Laplacian pyramid scheme. (a) analysis: one level of decomposition. The outputs are a coarse approximation  $a[n]$  and a difference  $b[n]$  between the original signal and the prediction. (b) the new proposed reconstruction scheme.



**Figure III.3.** Laplacian pyramid structure.

### ***III.3.2 Directional Filter Bank***

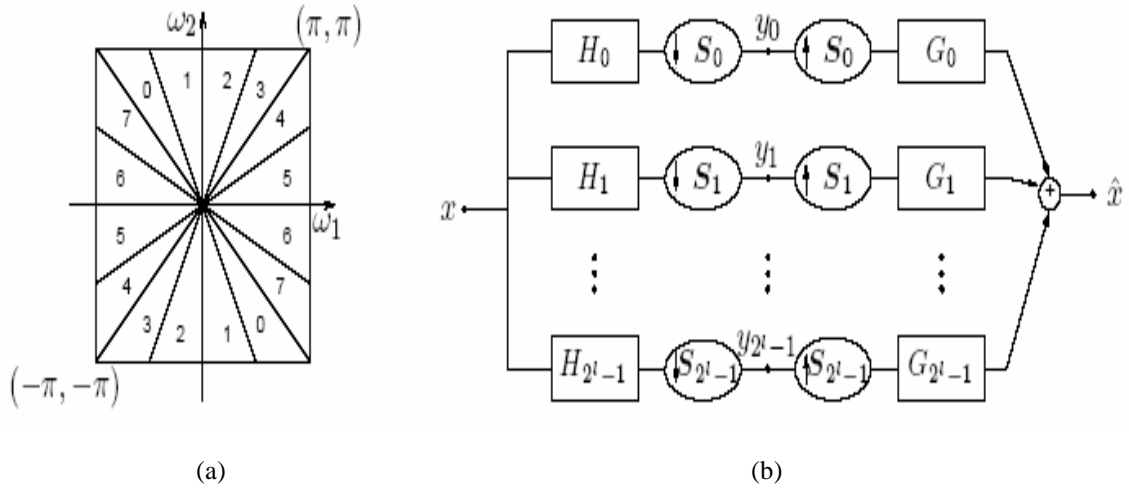
In 1992, Bamberger and Smith (*Bamberger & Smith, 1992*) introduced a 2-D directional filter bank (DFB) that can be maximally decimated while achieving perfect reconstruction. The directional filter bank is a critically sampled filter bank that can decompose images into any power of two's number of directions. The DFB partitions the frequency plane into a set of wedge shaped passband regions as shown in Fig.III.4(a). For an  $l$ -level decomposition, there are ' $2^l$ ' subbands with wedge-shaped frequency. The multidirectional analysis is achieved through using a tree structure implementation. Each stage in the tree consists of a two band filter banks that splits the input image into two subband images by two complementary fan filters (Fig.III.4(b)).

Do and Vetterli proposed a new construction for the DFB to avoid modulating input image involved by the original construction of the DFB, which we can obtain the desired 2D spectrum division. It has a simple rule for expanding the decomposition tree in which the DFB is intuitively constructed from two building blocks. The first building block is a two channel quincunx filter bank with fan filters as shown in Fig.III.5 that divides a 2D spectrum into two directions: horizontal and vertical. The second building block is a shearing operator which amounts to just reordering the image samples (*Do & Vetterli, 2005*).

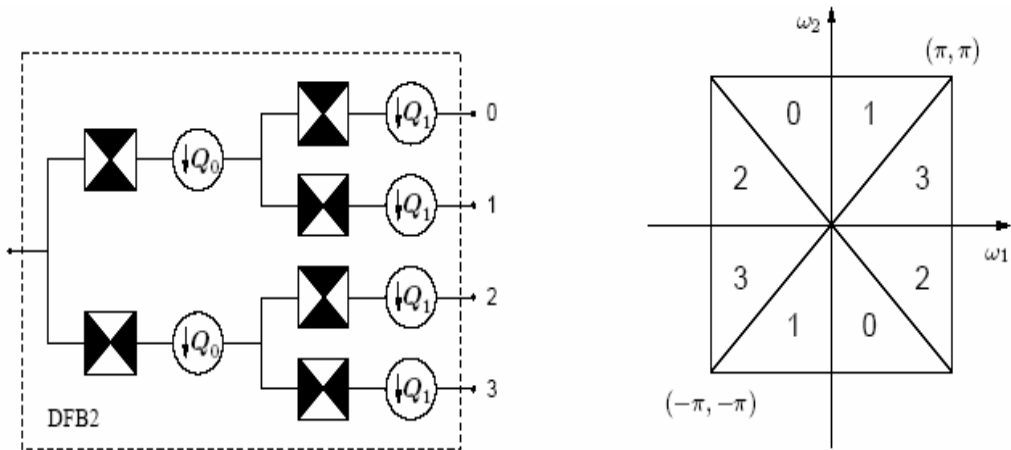
### ***III.3.3 Laplacian Directional Filter Bank***

The DFB is designed to capture the high frequency components of images (representing directionality). Therefore, low frequency components are handled poorly by the DFB. In fact, with the frequency partition shown in Fig.III.4(a), low frequencies would leak into several directional subbands, hence DFB does not provide a sparse representation for images. To improve the situation, low frequencies should be removed before the DFB. This provides another reason to combine the DFB with a multiresolution scheme.

Therefore, the LP permits further subband decomposition to be applied on its bandpass images. Those bandpass images can be fed into a DFB so that directional information can be captured efficiently.



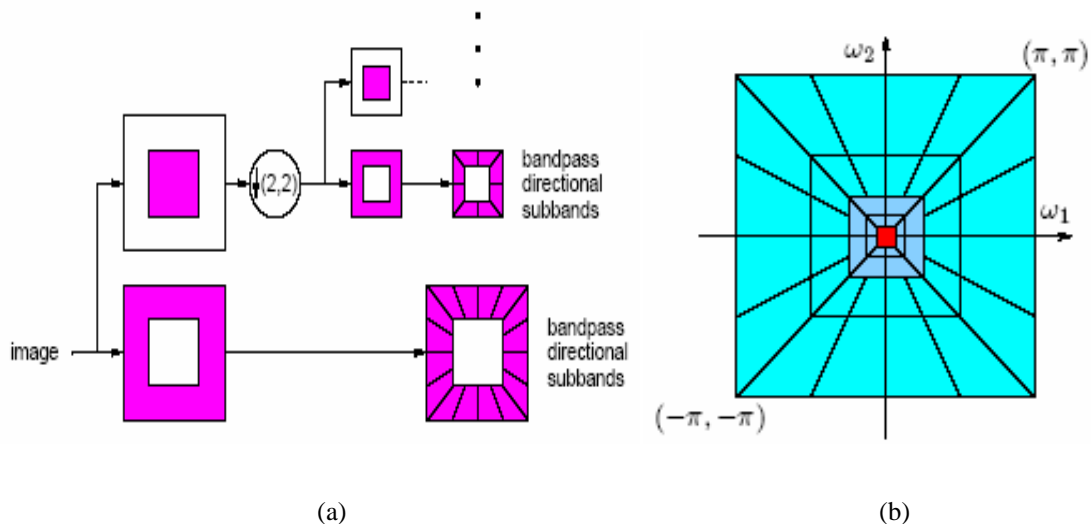
**Figure III.4.** Directional filter bank. (a) Frequency partitioning where  $l=3$ .  
 (b) The multichannel view of an  $l$  level tree structured DFB.



**Figure III.5.** Two-dimensional spectrum splitting using the quincunx filter banks with fan filters. The black regions represent the ideal frequency supports of each filter.

The scheme can be iterated repeatedly on the coarse image. The end result is a double iterated filter bank structure, named pyramidal directional filter bank (PDFB), which decomposes images into directional subbands at multiple scales. The scheme is flexible since it allows for a different number of directions at each scale.

In the newly constructed pyramidal directional filter bank, the Laplacian pyramid is first used to capture the point discontinuities, then followed by a directional filter bank to link point discontinuities into linear structures (Do & Vetterli, 2005). The overall result is an image expansion using elementary images like contour segments, and thus it is named the contourlet transform. The contourlet transform offers a flexible multiresolution and directional decomposition for images, since it allows for a different number of directions at each scale. Conceptually, the flow of operations can be illustrated by Fig.III.6(a) where the Laplacian pyramid iteratively decomposes a 2D image into lowpass and highpass subbands, and the DFB are applied to the highpass subbands to further decompose the frequency spectrum. Using ideal filters, the contourlet transform will decompose the 2D frequency spectrum into trapezoid-shaped regions as shown in Fig.III.6(b).

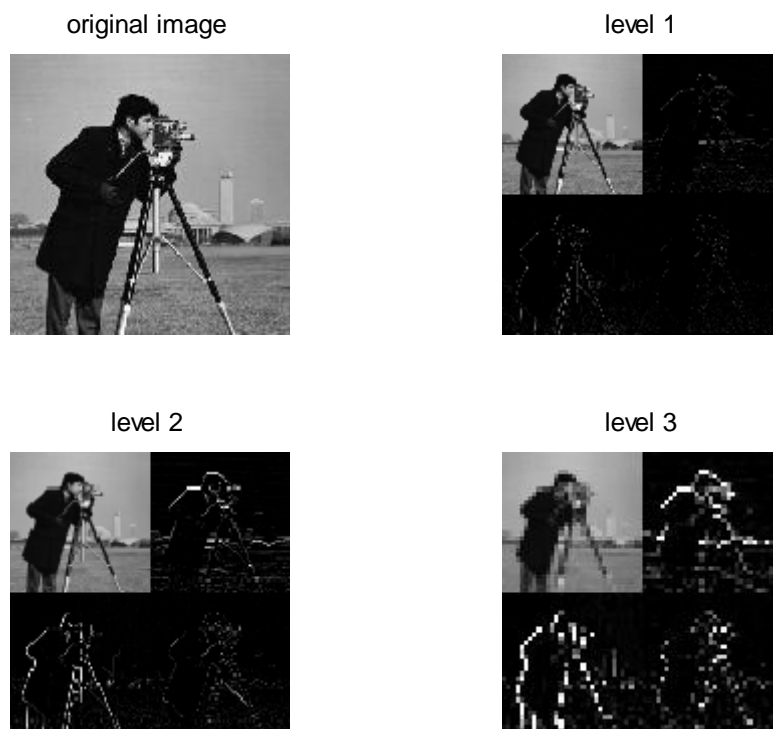


**Figure III.6.** The original contourlet transform. (a) block diagram. (b) Resulting frequency division.

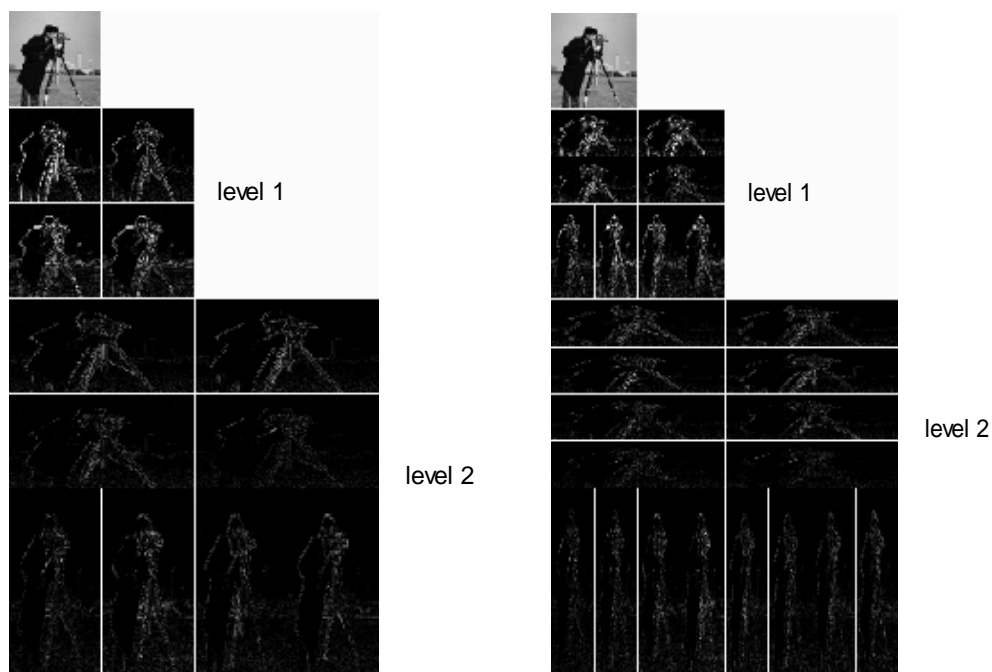
Figure III.7 illustrates an example of decomposition of the cameraman image using the wavelet and contourlet transforms. The wavelet decomposition is carried to the third level as shown in Fig.III.7(a). While the contourlet transform is presented with two examples of decomposition with two levels of the Laplacian pyramidal in each (Fig.III.8(b)). In the left, from the finer level with four directions to the coarse level with eight directions. In the right, the decomposition is doubled.

It can be seen that only contourlets that match with both location and direction of image contours produce significant coefficients. Thus, the contourlet transform effectively explores the fact, that the edges in images are localized in both location and direction. One can decompose each scale into any arbitrary power of two's number of directions, and different scales can be decomposed into different numbers of directions. This feature makes contourlets a unique transform that can achieve a high level of flexibility in decomposition while being close to critically sampled. Other multiscale directional transforms have either a fixed number of directions or are significantly over complete.





(a)



(b)

**Figure III.7.** Multiresolution decomposition of an image: (a) using wavelet transform, (b) using contourlet transform.

### **III.4 Nonsampled Contourlet Transform**

Due to downsampling and upsampling, the contourlet transform is shift-variant. However, shift-invariance is desirable in image analysis applications such as edge detection, contour characterization, and image enhancement (*Simoncelli and al., 1992*). To overcome this problem, a shift-invariance version of the contourlet transform is proposed by (*Cunha and al., 2006*) named Nonsampled contourlet transform (NSCT). The proposed transform can thus be divided into two shift-invariant parts: 1) a nonsub-sampled pyramid structure that ensures the multiscale property and 2) a nonsub-sampled DFB structure that gives directionality.

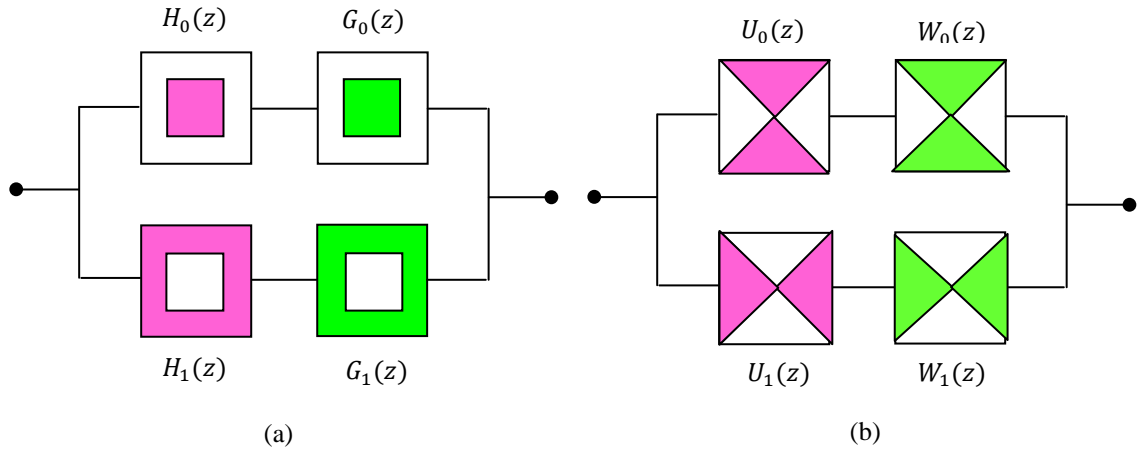
#### ***III.4.1 Nonsampled Pyramid (NSP)***

The nonsampled pyramid is completely different from the counterpart of the contourlet transform. The multiscale property of the NSCT is obtained from a shift-invariant filtering structure that achieves a subband decomposition similar to that of the Laplacian pyramid. This is achieved by using two-channel nonsampled 2-D filter banks as shown in Fig. III.8(a).

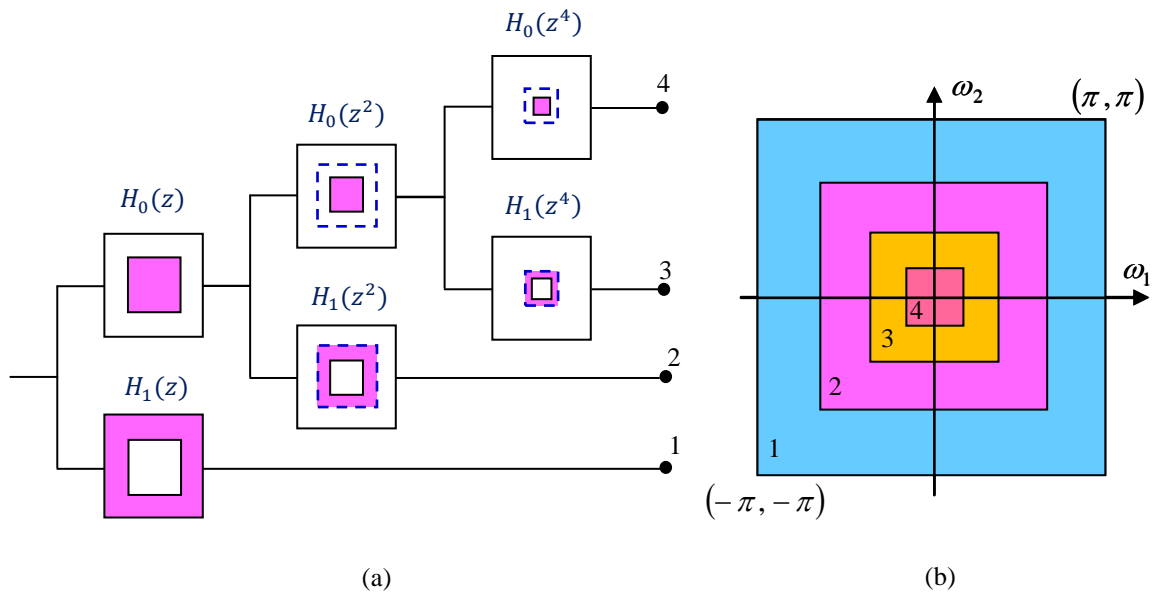
Specifically, the NSF is built from a low-pass filter  $H_0(z)$ . One then sets  $H_1(z)=1- H_0(z)$ , and the corresponding synthesis filters  $G_0(z)= G_1(z)=1$ , where  $G_0(z)$  and  $G_1(z)$  are low pass and highpass filters respectively. The perfect reconstruction condition is given as

$$H_0(z)G_0(z) + H_1(z)G_1(z) = 1 \tag{III.1}$$

The ideal frequency response of the building block of the nonsampled pyramid is given in Fig. III.8(a). To achieve the multiscale decomposition, we construct nonsampled pyramids by iterated nonsampled filter banks. For the next level, we upsample all filters by 2 in both dimensions. Therefore, they also satisfy the perfect reconstruction condition. Note that filtering with the upsampled filter  $H(Z)$  has the same complexity as filtering using the ‘a trous’ algorithm. The cascading of the analysis part is shown in Fig. III.9(a). These filters achieve multiresolution analysis as shown in Fig. III.9(b).



**Figure III.8.** Ideal frequency response of the building block of: (a) nonsubsampled pyramid; (b) nonsubsampled DFB.



**Figure III. 9.** Iteration of two-channel nonsubsampled filter banks in the analysis part of a nonsubsampled pyramid. (a) three stage pyramid decomposition. For upsampled filters, only effective passbands within dotted boxes are shown. (b) subbands on 2D plane frequency.

### III.4.2 Nonsampled Directional Filter Bank (NSDFB)

The directional filter bank given by (*Bamberger & Smith, 1992*) is constructed by combining critically-sampled two-channel fan filter banks and resampling operations. The result is a tree-structured filter bank that splits the 2-D frequency plane into directional wedges. A shift-invariant directional expansion is obtained with a nonsampled DFB (NSDFB).

The NSDFB is a shift-invariant version of the critically sampled DFB in the contourlet transform. The building block of a nonsampled DFB is also a two-channel nonsampled filter bank. However, the ideal frequency response for a nonsampled DFB is different, as shown in Fig. III.8(b).

To obtain finer directional decomposition, we iterate nonsampled DFB's. For the next level, we upsample all filters by a quincunx matrix given by

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (\text{III.2})$$

The frequency responses of two upsampled filters are given in Fig. III.10 and the cascading of the analysis part is shown in Fig.III.11.

The NSDFB is constructed by eliminating the downsamplers and upsamplers in the DFB. This is done by switching off the downsamplers/ upsamplers in each two-channel filter bank in the DFB tree structure and upsampling the filters accordingly. This results in a tree composed of two-channel NSFBS. Figure III.11 illustrates four channels of decomposition in which four direction frequency divisions are obtained. The higher level decompositions follow the same strategy, although they are more complex.

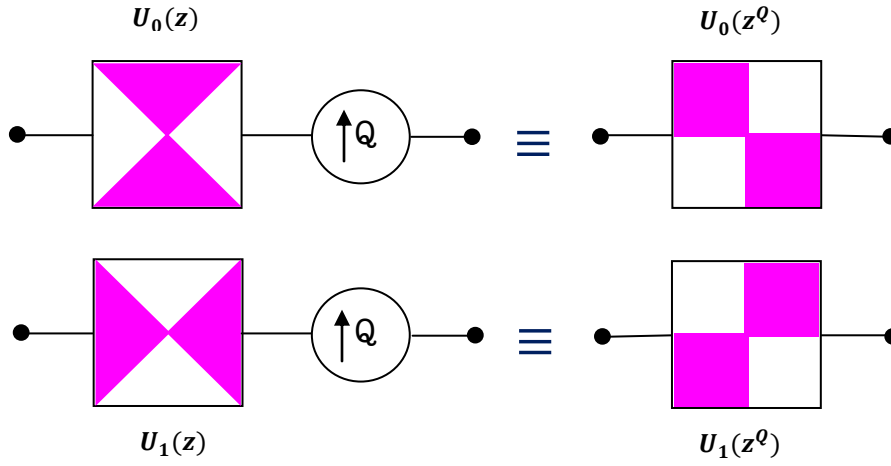


Figure III.10. Upsampling filters by a quincunx matrix  $Q$ .

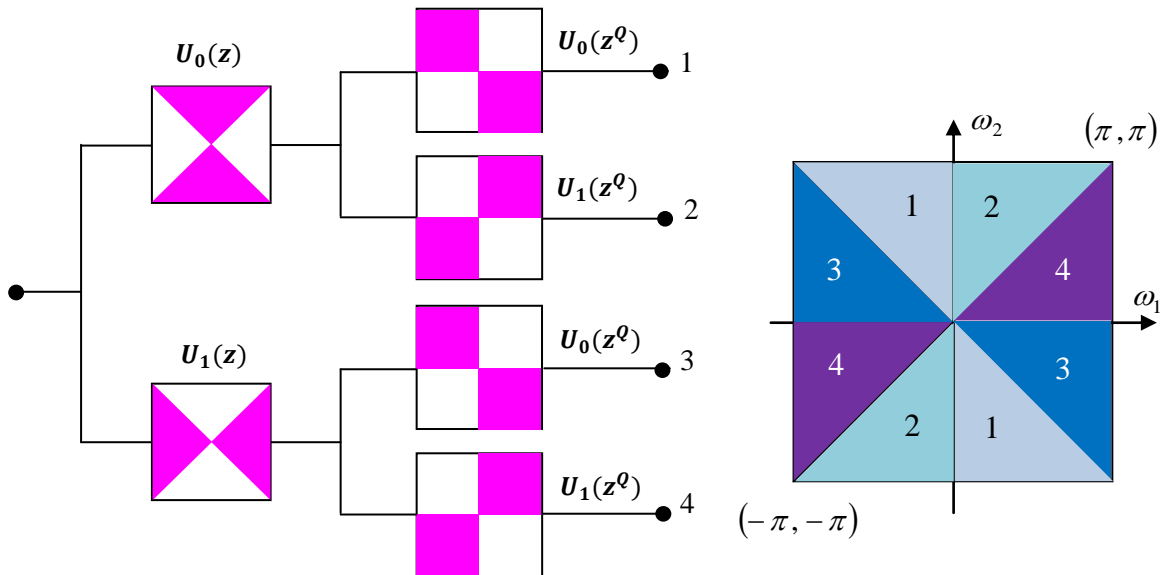
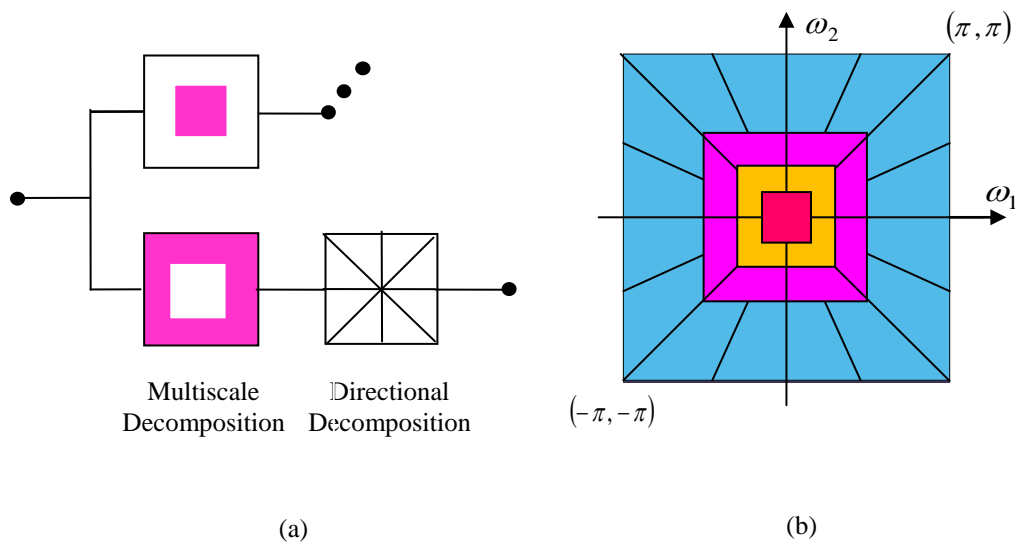


Figure III.11. The analysis part of an iterated nonsubsampling directional filter bank.

### III.4.3. Nonsampled Contourlet Transform

The NSCT combines the nonsampled pyramids which provide multiscale decomposition and nonsampled DFB's which provide directional decomposition as shown in Fig.III.12. First a nonsampled pyramid split the input into a lowpass subband and a highpass subband. Then a nonsampled DBF decomposes the highpass subband into several directional subbands. The scheme is iterated repeatedly on the lowpass subband outputs of the nonsampled pyramids. A nonsampled filter bank has no downsampling or upsampling, and hence it is shift-invariant.

The NSCT is built upon iterated nonsampled filter banks to obtain a shift-invariant directional multi-resolution image representation. Unlike separable transforms such as wavelets, the NSCT can efficiently capture the intrinsic geometric structures in natural images such as smooth contour edges and is fully shift-invariant, multiscale and multidirection expansion that has a fast implementation. An example of an image using NSCT decomposition is shown in Fig.III.13.



**Figure III.12.** The nonsampled contourlet transform: (a) Block diagram. (b) Resulting frequency division, where the number of directions is increased with frequency.

Nonsampled Contourlet coefficients level 1



NSSC coefficients: level 2



NSSC coefficients: level 2



NSSC coefficients: level 3



NSSC coefficients: level 3



**Figure III.13.** Example of decomposition of cameraman image using NSCT with two level of Laplacian pyramid with two subbands directions in each level. The finer level contains the LL subbands.

### **III.5 Conclusion**

In this chapter, we have introduced the main multi-resolution transforms used recently in the field of image processing. The multi-resolution scheme provides a simple hierarchical framework for interpreting the image information.

The standard two-dimensional wavelet transform is widely used in image processing. The major drawback for wavelets in two-dimensions is their limited ability in capturing directional information. To overcome this deficiency, researchers have recently considered multiscale and directional representations that can capture the intrinsic geometrical structures such as smooth contours in natural images. The contourlet transform is a directional transform, which is capable of capturing contours and fine details in images. It has better performance in representing the image salient features such as edges, lines, curves and contours than the wavelet transform because of its anisotropy and directionality.

Compared to the contourlet transform, the NSCT is a fully shift-invariant, multi-scale, and multi-direction image decomposition that has better frequency selectivity and regularity, and a fast implementation. Consequently, the primary motivation of this thesis is to exploit the NSCT properties for image registration.



# **PART B**

## **CONTRIBUTIONS CHAPTERS**

# CHAPTER IV

## **A WAVELET AND GA BASED IMAGE REGISTRATION TECHNIQUE**

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## IV.1 Introduction

The key idea of the image registration process is focused on determining the unknown parametric *transformation*, which relates both images, by placing them in a common coordinate system bringing the points as close as possible. Because of the uncertainty underlying such transformation, the image registration task arises as a *non-linear problem* that cannot be solved by a direct method (e.g. resolution of a simple system of linear equations). It should be solved by means of an iterative procedure searching for the *optimal estimation* following a specific search space optimization scheme aiming at minimizing the error of a given *Similarity metric* of resemblance. The figure IV.1 describes the framework of the image registration process (Cordón and al., 2003).

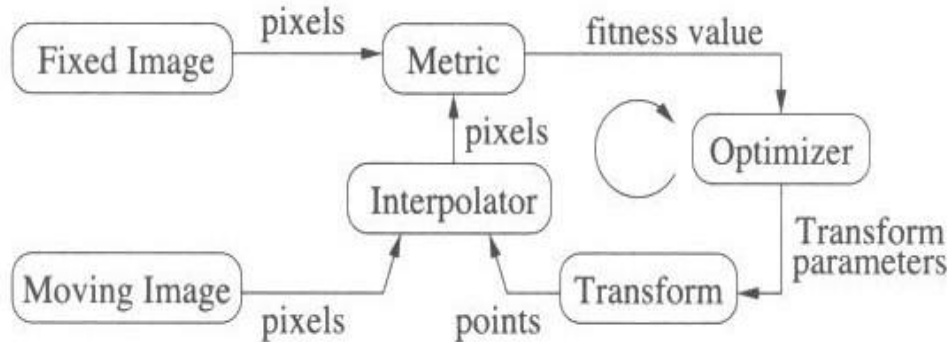


Figure IV.1. Image registration framework.

Classical local optimizers can be used for this task although their main drawback is that they usually get trapped in a local minima solution. The main reasons for such behavior are related to both the nature of the problem to be tackled and the greedy/local search features of these methods. Hence, the interest on the application of evolutionary algorithms to the image registration optimization process has increased in the last decade due to their global optimization nature.

A lot of researchers have attempted to apply GAs to help search over the complex search landscape in image registration. The genetic search can adequately handle simple translations and rotations in the image registration task, even in the presence of variation in contrast (Fitzpatrick and al., 1984).

The advantage of GAs has also been tested in a complex and noisy environments by (Xin and al., 2002) to search for the position and orientation of target image for object recognition. An overview of several researches and techniques of applying GA to image registration is provided in the next chapter. In the following part of this chapter, we present a hybrid GA method which is proposed to maximize the robustness of the search technique in order to provide better quality in registration.

#### **IV.4 Hybrid GAs Techniques**

As they use the fitness function only in the selection step, GAs are blind optimizers which do not use any auxiliary information such as derivatives or other specific knowledge about the special structure of the objective function. If there is such knowledge, however, it is unwise and inefficient not to make use of it. Several investigations have shown that a lot of synergism lies in the combination of GAs and conventional methods.

Many variations of the standard GA can be found in the literature that are grouped in three classes:

1. The GA performs coarse search first. After the GA is completed, local refinement is done.
2. The local method is integrated in the GA. For instance, every  $K$  generations, the population is doped with a locally optimal individual.
3. Both methods run in parallel: All individuals are continuously used as initial values for the local method. The locally optimized individuals are re-implanted into the current generation.

All these modifications and hybridizations have been motivated by a desire to improve the performance of the GA, and to adapt it to particular problem domains. The proposed hybrid GA is based on two techniques namely fitness sharing and elitism techniques.

### IV.2.1 Fitness Sharing Technique

The simple GA is able to explore effectively a multimodal search space. However it tends to find one single optimum, thus it can still be trapped in local optima. This problem is the result of *genetic drift*, which a tendency of GA to select a population with similar chromosomes, thus to converge towards one solution. One strategy to overcome this problem consists in maintaining population diversity, so that different sub-populations are able to explore different portions of the search space, in order to identify and converge towards different multiple optima. Niche based GA represent an elegant and nature inspired solution to address the issue of keeping the population diversity (*Giuseppe & Luigi, 2007*). An analogy to multimodal domains exists in nature in the form of "niches" which are subspaces that can support different types of species or organisms. The fertility of each niche as well as the efficiency of each organism at exploiting niche fertility affects the number of organisms within each niche. This has inspired the consideration of each peak in a multimodal domain as a niche in the framework of what has come to be called niche formation methods. The two most popular niche formation methods are *fitness sharing* and *crowding*. Niching methods have been developed to minimize the effect of genetic drift resulting from the selection operator in the traditional GA in order to allow the parallel investigation of many solutions in the population (*Gao & Hue, 2006*). In this work, we have employed the sharing technique.

The idea behind the sharing method is to reduce the fitness of individuals that are very similar in their chromosome. In this way, individuals that uniquely exploit portions of the search space are privileged for reproduction while discouraging redundant individuals in the same area (*Giuseppe & Luigi, 2007*). Fitness sharing forces the development of more than one niche in a given population. It is based on the idea that individuals in a particular niche have to share the available resources. The more individuals are located in the neighborhood of a certain individual, the more its fitness value is degraded. Mathematically, the shared fitness  $f_i'$  of individual  $i$  with fitness  $f_i$  is defined as follows

$$f_i'(x_i) = \frac{f_i(x_i)}{\zeta_i} \quad (\text{IV.1})$$

Where  $\zeta_i$  is the niche count which is an indication of how crowded an individual's niche is. Consequently, points in a dense cluster will have a lower fitness value and will be less likely to be passed on to the next generation.  $\zeta_i$  is defined as follows

$$\zeta_i = \sum_{j=1}^n Sh(\Delta_{ij}) \quad (IV.2)$$

$\Delta_{ij}$  is the Euclidian distance between two individual  $i$  and  $j$ . A power law function is commonly used for a sharing function, as depicted in equation (IV.3)

$$sh(\Delta_{ij}) = \begin{cases} 1 - (\Delta_{ij}/\sigma_s)^\alpha & \text{if } \Delta_{ij} < \sigma_s \\ 0 & \text{otherwise} \end{cases} \quad (IV.3)$$

$\alpha$  is a heuristic parameter set by the user, used to regulate the shape of the sharing function. When  $\alpha = 1$ , the function is triangular.  $\sigma_s$  is a parameter called the *niche radius* set by the user as the minimal separation desired or expected between the solutions.  $\sigma_s$  defines the size of a niche, so that if the distance between two points is less than  $\sigma_s$ , the fitness value for those points is reduced. Fitness sharing in objective space is used to find a uniformly distributed non-dominated solution set. The effect of this scheme is to encourage the search process in unexplored regions.

### ***IV.2.2 Elitism Technique***

In the basic GAs, it is possible for the next generation to have a best individual with a lower fitness than the best individual encountered in a preceding generation. This loss of the best individual occurs due to the probabilistic nature of the GA selection, crossover and mutation, and hence helps to improve the overall performance of the algorithm. To overcome this problem, we use the elitism technique. It is an effective tool to improve the performance capability of GAs, because it prevents losing the best found solutions by conserving the best solutions obtained in the optimization process.

Figure IV.2 shows the details of the proposed hybrid GA scheme.

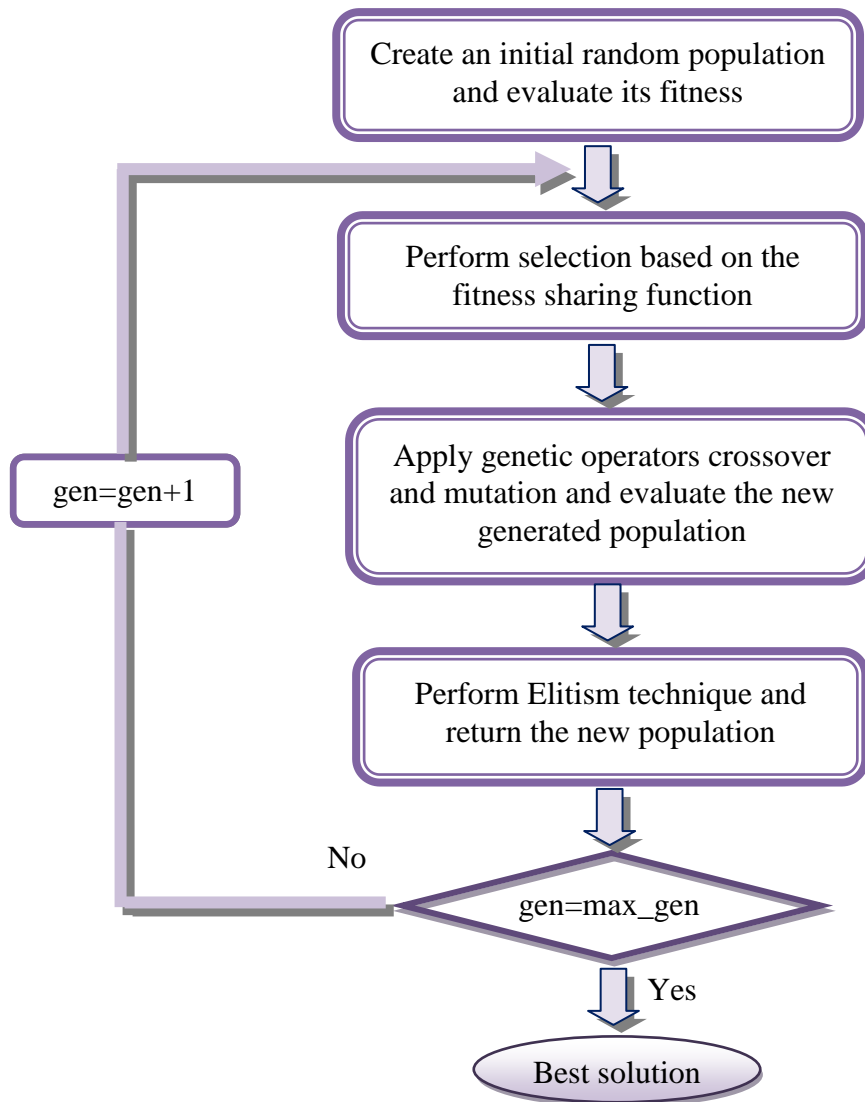


Figure IV.2. A hybrid GA cycle.

### IV.3 GAs based Image Registration

For the GAs to be successful, how to formulate the chromosome and fitness function is very important. The GAs will have a better convergence behavior if the fitness function is generally continuous and the chromosome with the optimal fitness value corresponds to the target solution. In the following, formulations of the chromosomes and the fitness function for image registration are described.

#### IV.3.1 Chromosome Encoding

In this thesis, for 256x256 images, a binary string is adopted to represent a chromosome for rigid transformation. The chromosome string is composed of three genes. The gene R represents the rotational transform, the gene X represents the x-axis translational transform, and gene Y represents the y-axis translational transform as shown in Figure IV.3.

An 8-bit field is used to represent the possible relative rotation of the input image to the reference image; and 6 bits are used to express the translation in the x-axis and the y-axis. Thus the length of each chromosome is 20 bits. All representations are signed magnitude, using one bit for the sign and the rest of the bits to represent the magnitude of the rotation or translation. Thus, the relative rotation has a range of  $\pm 127$  degrees, while the relative translation in the x (or y) direction has a range of  $\pm 31$  pixels.

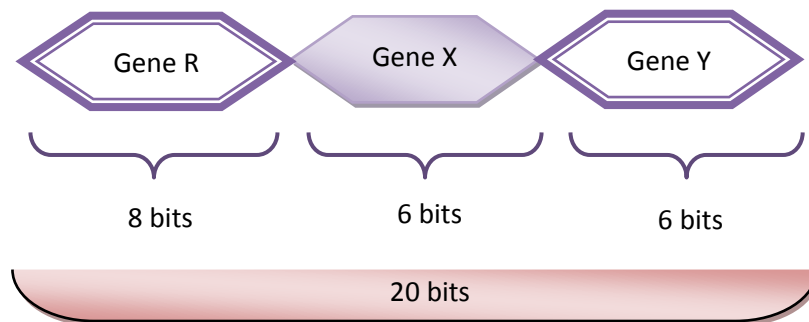


Figure IV.3. Chromosome encoding scheme.



### IV.3.2 Objective Function

To measure optimality, a fitness function can be used. This fitness function provides a numerical measure of the goodness of a proposed answer of the registration problem. The validation of registration is measured by the correlation coefficient between two aligned images. Hence, the correlation coefficient method can be used as an objective function which has to be optimized to a maximum value. Given two images A and B, the correlation coefficient is defined in equation IV.4 as

$$C(A, B) = \frac{\sum_{i=0}^{N-1} \sum_{j=0}^{N-1} ((A_{ij} - \bar{A}) \times (B_{ij} - \bar{B}))}{\sqrt{\sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (A_{ij} - \bar{A})^2 \times \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (B_{ij} - \bar{B})^2}} \quad (IV.4)$$

Where A and B are images of size N x N, while  $A_{ij}$  and  $B_{ij}$  denote pixel values in A and B respectively.

## IV.4 Simulation Results and Discussion

For this problem, considering the range of transformation value that we are going to find. The rotation values ranged from -127 to 127 degrees while the displacement of x-axis and y-axis values are considered between -31 to 31 pixels. Every individual represents a combination of all transformation parameters which describe an image transformation. The domain of search for these parameters is large ( $255 \times 63 \times 63 = 1012095$ ) which is suitable for GAs to explore.

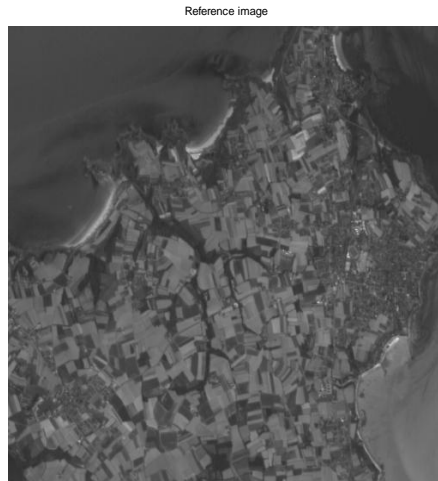
### GAs Parameters:

Population size: 80 individuals  
Crossover type: one point crossover  
Crossover rate: 0.85  
Mutation type: binary mutation  
Mutation rate: 0.02  
Generation gap: 100

**Example 1: Application to SPOT images**

We have applied our registration algorithm (*Meskine & Taleb, 2007*) based on GAs to SPOT images for which the reference image is a SPOT panchromatic image of the Mont Saint-Michel region (France) as shown in Fig.IV.4.

We have adopted a rigid transformation which uses three parameters, rotation R and translations in X and Y directions. The second image named transformed image to be compared with the reference image is displaced by (X=13, Y=9) from the center of the first and rotated by R=7 degrees. These tested images with these initial parameters are also considered in the next simulations.



**Figure IV.4.** A SPOT panchromatic image of the region of Mont Saint-Michel (France).

The simulation results are as follows: Figure IV.5 illustrates the best solution trajectory at the GA run. *bestf* is the best fitness value which measures the maximum correlation coefficient obtained at each generation. Higher maximum fitness gives better accuracy in estimating the transformation value. The GA requires between 20 to 30 generations to converge and to yield a near correct estimation. Figure IV.6 depicts the evolution of the parameters (R,X,Y) during the generations. The red dashed lines show the initial parameters and blue lines show the optimal parameters found during the run the GAs. We see that the optimal parameters values are closer to the initial parameters.

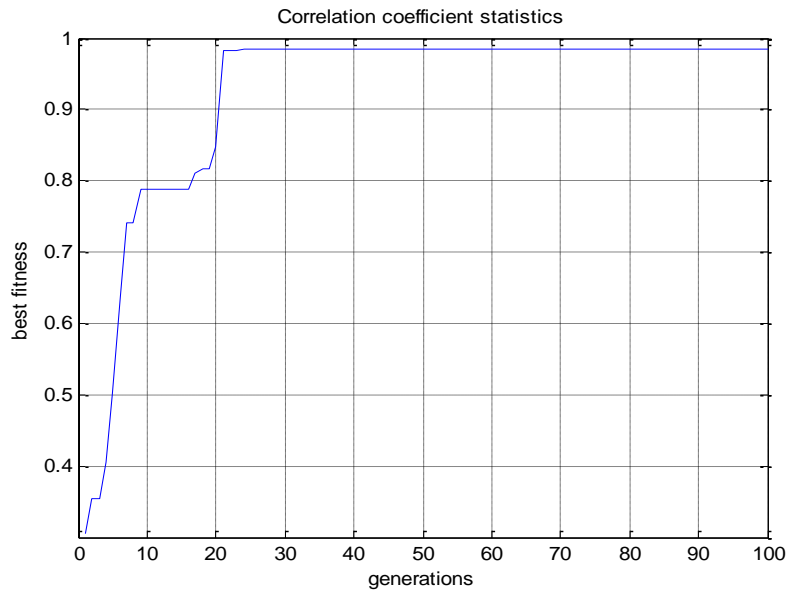


Figure IV.5. generations vs optimum fitness for GA.

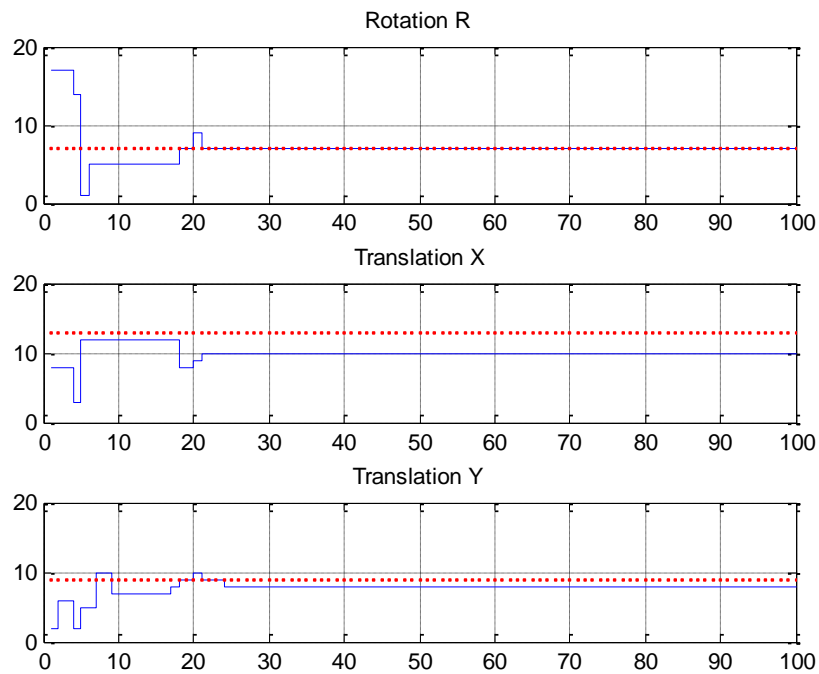


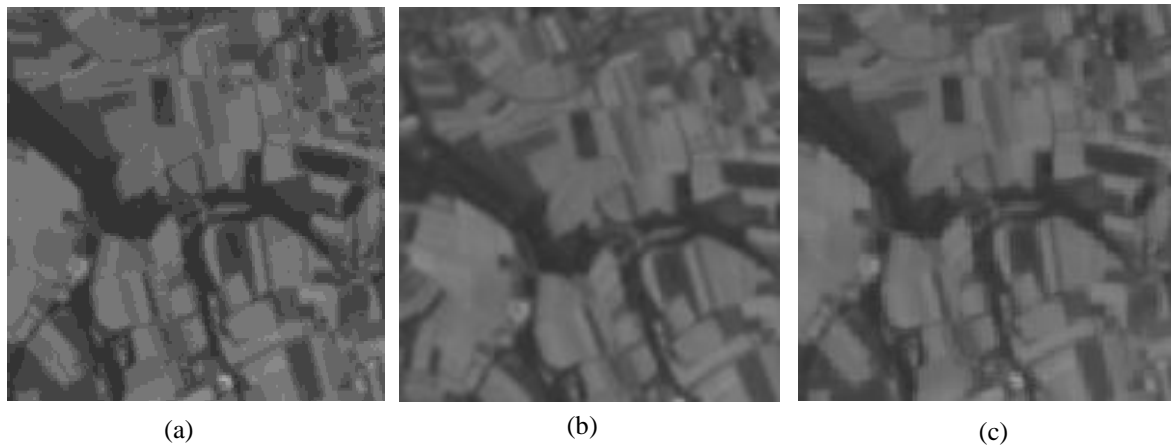
Figure IV.6. Evolution of the transformation parameters (R,X,Y) during the generations

Table IV.1 gives the analytical results of registration. The full time needed for reaching the 100 generations is about of 1602.813000 seconds.

<b>Transformation parameters</b>	<b>R (degrees)</b>	<b>X (pixels)</b>	<b>Y (pixels)</b>
The true parameters	7	13	9
The obtained parameters	7	10	8

**Table IV.1.** Comparison of the obtained transformation parameters using the proposed registration algorithm with the true/known ones.

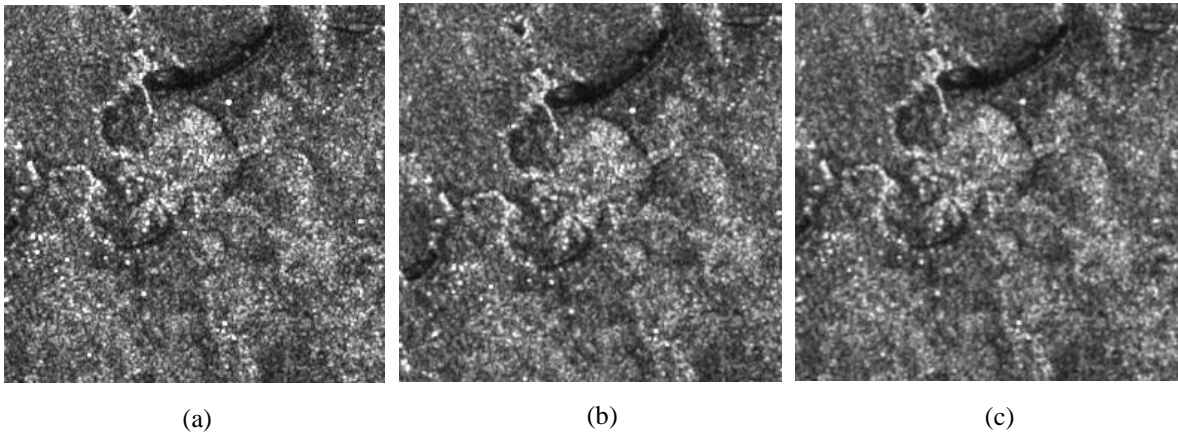
The two images to be compared, reference and transformed images, and the resulting registered image obtained using the proposed registration algorithm based on GAs are shown respectively in Fig.IV.7 with a size of 128\*128 pixels.



**Figure IV.7.** Registration of satellite images: (a) Reference image, (b) Transformed or input image to be corrected, (c) Registered image.

**Example 2: Application to SAR images**

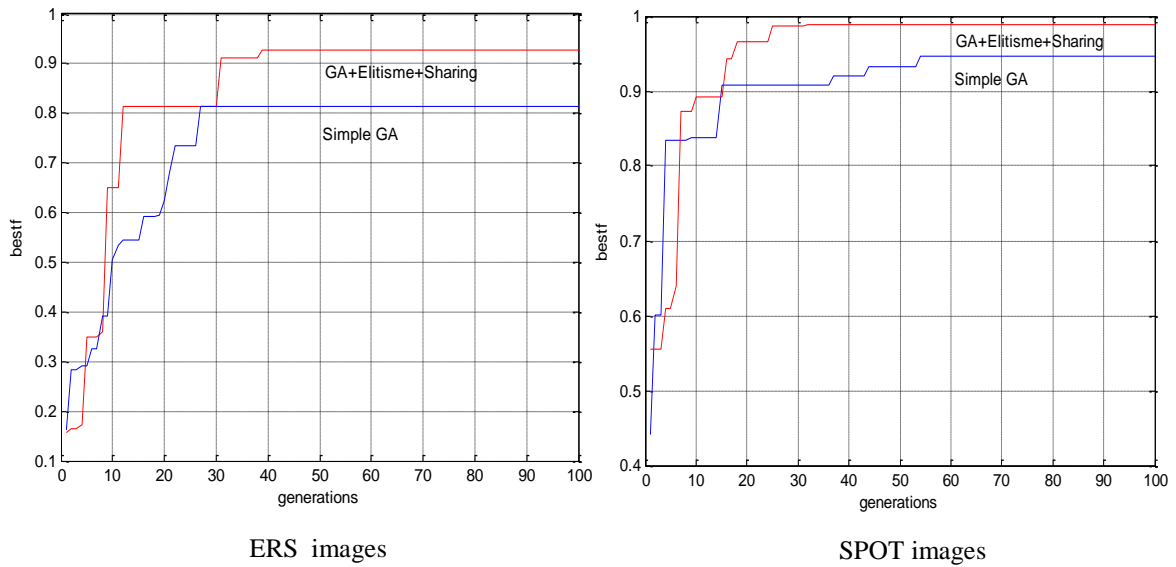
For a second test, we have applied our registration algorithm to Synthetic Aperture Radar (SAR) images which present a significant noise. Figure IV.8 illustrates the two images to be compared of size (256\*256) pixels with the registered image result. This test is conducted with the same parameters as previously and we have found the same obtained transformation parameters. Therefore, this approach is clearly a promising registration method.



**Figure IV.8.** Image registration results of SAR images using proposed GA: (a) Reference image, (b) Transformed image to be aligned, and (c) the registered image.

To show the performance of the hybrid GA which combines elitism and fitness sharing techniques, we have first performed our algorithm with a simple GA. For the sharing technique, we have used the parameters  $\sigma_s = 1.2$  and  $\alpha = 1$ , and for elitism we have conserved 5% individuals of the population at each generation.

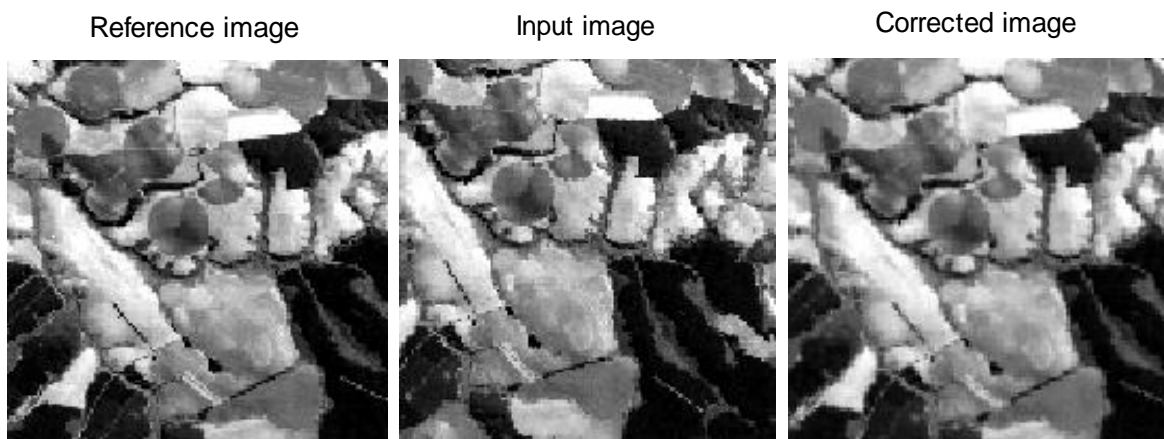
Figure IV.9 illustrates the evolution of the best solution during generations. The parameter ‘bestf’ indicates the fitness of the best individual (maximum fitness) obtained during the run of GAs. It is clear that the higher maximum fitness gives better accuracy in estimating the optimal value, which is indeed obtained by the hybrid GA compared to the simple GA. The test algorithm is applied for both SAR and SPOT images.



**Figure IV.9.** Evolution of best solution during the run of the proposed GA marked with red line and the simple GA marked with blue line in the cases of SPOT and ERS images.

**Example 3: Real Application to multi-temporel Landsat-TM images**

To demonstrate the robustness of our algorithm, we have implemented it for registering two images taken from the Landsat-TM sensor at different dates from an agricultural region of Sao Paulo (*Meskine and al., 2009*). The reference and input images to be compared as well as the resulting image are shown respectively in figure IV.10.



**Figure IV.10.** Registration results of the multi-temporal agricultural images using our algorithm of registration based on GAs.

## **IV.5 Wavelet Transform for Image Registration**

Exhaustive search is computationally expensive and the computational cost increases exponentially with the number of transformation parameters and the size of the data set. The multi-resolution scheme presented in (*Mallat, 1989*) represents a very fast algorithm for on-line processing of imagery data.

When using such multi-resolution data, the size of the search data can be reduced by initially searching at lowest resolution and then proceeding to higher resolution images where the search results are only refined (*LeMoigne, 1995*) and (*Corvi & Nicchiotti, 1995*).

The wavelet transform presents a powerful tool for multi-resolution analysis. It can be used to improve the image registration accuracy by considering both spatial and spectral information and by providing multi-resolution representation to avoid losing any global or local information (*LeMoigne, 1994*). Additional advantages of using the wavelet-decomposed images include bringing data with different spatial resolutions to a common resolution using the low frequency sub-bands while providing access to edge features using the high frequency sub-bands (*LeMoigne, 1997*). A lot of work in image registration has focused on the use of wavelet transform, a citation is given in the related work part of the next chapter.

Applying registration in this fashion results in two major advantages. First, the number of pixels is reduced at coarser levels, which simplifies the computations. This results in large computational gains since most of the search iterations are usually executed in the coarser levels. Second, by down sampling and successive smoothing of the image, only large-scale features are preserved, causing the similarity metric to be computed on smoother images. This property can prevent the search algorithm to get stuck in local minimums in the search space.

In this part, we present a registration method for satellite images based on GAs within a multi-resolution framework based on the wavelet decomposition (*Meskine and al., 2009A*).

### ***IV.5.1 Registration Strategy***

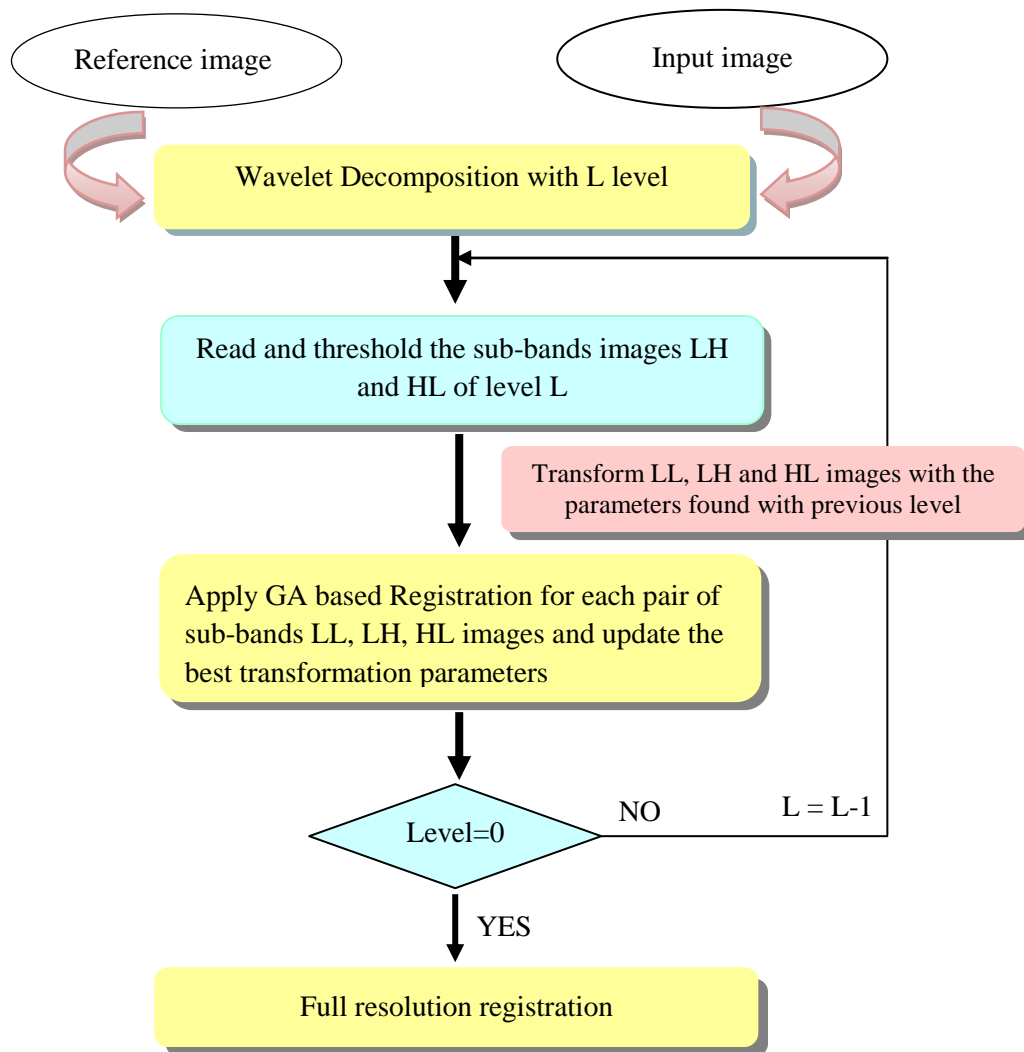
In order to achieve computational efficiency, the search strategy follows the multi-resolution decomposition, working iteratively from the deepest level of decomposition (where the image size is the smallest) to the top level of decomposition, i.e going from coarse to fine spatial resolution.

First, both the reference and input images are decomposed following a multi-resolution wavelet decomposition. For each level of decomposition, three images are obtained, namely LL, LH and HL. The sub-band HH includes the high frequency noise which affects image matching, and is, therefore, not useful for registration. Also, the LH and HL wavelet-compressed images of the input and the reference images are thresholded to remove noise and extract edge features (*Chalermwat & El-Ghazawi, 1999*).

At each level of the decomposition, the search focuses on an interval around the “best” transformation found by the GA at the previous level and is refined at the next level up. The correlation ratio between sub-band images of the reference image and the input image is successively computed and maximized. The best transformation parameters found with the GA are used to correct the subbands of the next level; and therefore the search technique of new optimum parameters is conducted until the full resolution.

The flow chart of the algorithm of registration based on the wavelet transform is illustrated in Fig.IV.11.





**Figure IV.11.** Framework of registration algorithm based on the wavelet transform and GA.

### **IV.5.2 Simulation Results**

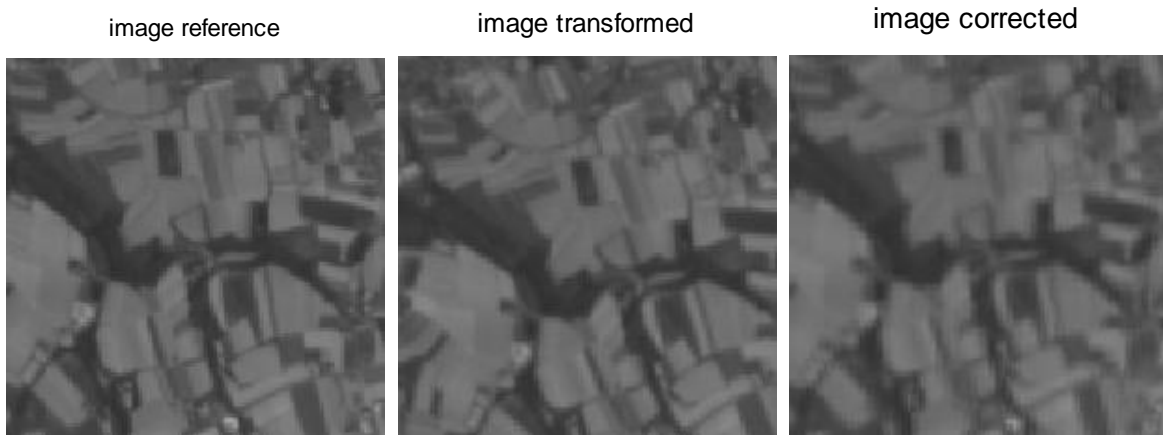
We have applied our registration algorithm based on the GA and the wavelet transform on SPOT satellite images for which the tested images are described as before. In this simulation, we have limited the decomposition level to three and we have used the Haar wavelets which is easy to implement.

The results obtained for the correlation coefficient from each sub-band at each level are depicted in Table IV.2. We can see that the correlation coefficient computed for each sub-band increases from level to another from the deepest to the full resolution. Therefore, the task of the multi-resolution strategy is completed.

<b>level</b>	<b>Sub-bands</b>	<b>Correlation coefficient</b>
<b>1</b>	LL	<b>0.9757</b>
	LH	<b>0.7440</b>
	HL	<b>0.3815</b>
<b>2</b>	LL	<b>0.9645</b>
	LH	<b>0.5445</b>
	HL	<b>0.2883</b>
<b>3</b>	LL	<b>0.9673</b>
	LH	<b>0.5417</b>
	HL	<b>0.1398</b>

**Table IV.2:** Correlation coefficient for different sub-bands.

The corrected image obtained with this method is illustrated in Fig.IV.12 with pair images to be compared (reference and transformed) of size 128\*128 pixels.



**Figure IV.12.** Registration of satellite images using the wavelet transform and a GA.

## **IV.6 Conclusion**

The goal of the registration task is to find the transformation that best represents the relative transformation between two compared images. Unlike the traditional linear search, GAs adaptively explore the search solution space in a hyper-dimension fashion, so that they can improve computational efficiency. Image registration can be regarded as an optimization problem, where the goal is to maximise a measure of image similarity. Recent works have investigated the global search technique as a GA to find the best transformation parameters.

The GA framework has been tuned optimally to find the most accurate values of transformation with low computational time that is suitable for efficient real-time inspection environment. Therefore, a hybrid GA was proposed. This method outperformed classical GAs significantly because the method benefited from both specialties of elitism and fitness sharing when it successfully achieved an excellent result.

Also, we have presented a registration method for satellite images that uses multi-resolution wavelet decomposed images to reduce the search data, and GAs to optimize the search solution space. The simulation results have shown the effectiveness of the proposed method for registering satellite images which considers the three parameters of transformation (rotation, and x-axis and y-axis translations).

# CHAPTER V

## ***AN NSCT & GA BASED IMAGE REGISTRATION TECHNIQUE***

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## **V.1 Introduction**

A wide range of registration techniques has been developed for many different types of applications and data. Given the diversity of the data, it is unlikely that a single registration scheme will work satisfactorily for all different applications. Some techniques are proposed to find a geometrical transformation that relates the points of an image to their corresponding points of another image.

To achieve fast execution, the search technique must be very efficient. The key to this is to employ advanced search methodologies that avoid exhaustively examining all possible solutions in the search space, by rendering parts of the search space useless. Genetic algorithms and multi-resolution searches are two attractive techniques that can efficiently explore the huge search scope of registration.

Registering images in a multi-resolution way is a frequently used measure which can improve the result and decrease the complexity of the algorithm. A similar effect can be achieved by combining different techniques at certain levels of the registration procedure. Different transformation models or different search strategies can be combined to produce one final registration result. Applying registration in this fashion results in two major advantages. First, the number of pixels is reduced at coarser levels, which simplifies the computations. This results in large computational gains since most of the search iterations are usually executed in the coarser levels. Second, in successive smoothing of the image, only large-scale features are preserved, causing the similarity metric to be computed on smoother images. This property can prevent the search algorithm to get stuck in local minimums in the search space.

The shift variance is very important in image registration, since it affects the geometric accuracy of the registered image. NSCT is a fully invariant transform which provides a multiresolution and multidirectional representation of the image. Hence, our motivation is to exploit the NSCT properties for image registration. In this chapter, we present a registration technique in which we apply the NSCT approach using GAs for registration of satellite images. The main idea is to combine local search methods with global ones balancing exploration and exploitation, to speed up the search of the best transformation parameters.

## **V.2 Related Work**

Most of the previous work in image registration has focused on the use of wavelets. The wavelet transform features are used because wavelet transforms convey both space and time characteristics and their multi-resolution representations enable efficient hierarchical searching. LeMoigne *et al.* presented a cross-comparison of automated registration algorithms for multiple source remote sensing data in (*LeMoigne, 1997*), in which a multi-resolution wavelet-based (MRW) image registration was used. The algorithm requires no *a priori* knowledge in order to perform automatic registration. The similarity measure is based on the normalized cross-correlation. The work in (*Pinzon and al., 1997*) proposed automatic wavelet-based image registration based on point matching techniques. Unlike LeMoigne's technique, the similarity measure is based on automatically extracted control points from the wavelet-compressed images. The registration result is determined by matching these control points. However, using similar test data as in (*LeMoigne, 1997*), poor registration accuracy results were reported. The translation invariant wavelets and their application were explored in (*Chettri and al., 1997*). An image registration algorithm using translation invariant wavelets was developed. However, the study produced poor results and concluded that the translation invariant wavelets have limited applications in image registration. Fonseca and Manjunath presented a multi-resolution registration that relies on the grey level information content of the images and their local wavelet transform modulus maxima (*Fonseca and al., 1997*). The proposed algorithm consists of five major steps: (1) smoothing the image, (2) decomposing the image using wavelets, (3) extracting the feature points, (4) matching the feature points, and (5) refining the matching in higher resolutions. The registration error is less than one pixel for noise-free images. However, only clean images were used. The registration might fail if the images contain anomalies. The work in (*William & Karl, 2003*) investigates the use of wavelets to automatically register remotely sensed images. The proposed algorithm is based on the Laplacian of Gaussian filter to automatically extract ground control points and the discrete wavelet transform for multi-resolution analysis. The inherent multi-resolution processing of the image data provides an efficient method for registering large image data sets.

Image registration can be regarded as an optimization problem, where the goal is to find the best transformation parameters which maximize the measure similarity between compared images. GAs have been known to be a robust technique for search and optimization problems. Unlike traditional linear searches, GAs adaptively explore the search solution space in a hyper-dimensional fashion so that they can improve computational efficiency. Numerous researchers have attempted to apply GAs to help search over the complex search landscape in image registration.

Fitzpatrick was one of the first to investigate the applicability of GAs for image registration (*Fitzpatrick and al., 1984*). His work focuses on medical images obtained by X-ray, gamma ray, and imaging (NMR). Experiments were carried out using simulated data, however neither accuracy nor computing performance was quantified. In order to improve the performance, Ozkan (*Ozkan and al., 1989*) proposed a parallel implementation of Fitzpatrick's GA but no algorithmic analysis or performance evaluation was quantified. Dasgupta and McGregor (*Dasgupta & McGregor, 1992*) proposed a structured GA for automatic registration organized in two levels: the higher level activates or deactivates sets of lower level genes. Although this GA is claimed to be five times faster than the Fitzpatrick algorithm, neither quantitative accuracy measurements nor quantitative computing performance were presented. Turton and Arslan reported a hardware VLSI based design of their parallel GA registration (*Turton and al., 1994*). The image registration is done in a compressed domain using the discrete cosine transform. The coefficients found under transformation have some limitations in their implementation. The work in (*Ou and al., 1996*) proposes a high speed image registration algorithm that determines the frame-to frame translational motion in an image sequence. A modified GA and the sequential similarity detection algorithm are used to achieve fast registration. The problem with this method is that the chromosome length is very limited and the similarity measure is not as good as the normalized cross-correlation. Maslov and Gertner (*Maslov & Gertner, 2001*) proposed to include a gradient analysis of the fitness function within the GA iteration, in order to better drive the search space exploration. Experimentation shows that this approach can increase the efficiency of GAs when they are applied to an image registration problem. Laksanapanai (*Laksanapanai and al., 2005*) proposed a parallel implementation of GA based intensity image registration using MPI (Message Passing Interface). The application

field is medical image registration. The result for multi-modality alignment is very promising.

All these works have attempted to apply either wavelets or GAs, but fewer works have combined these two approaches in one technique, as those of Chalermwat and Ghazawi (*Chalermwat & El-Ghazawi, 1999*) who utilize the multi-resolution property in a wavelet domain and GA to reduce the search data size as well as the search scope. Using a coarse to fine grain scheme, multi-resolution techniques reduce the size of the search data by searching initially at the lowest resolution first and then proceed to higher resolutions where the search results are only refined. For each level, the best result found from the previous level is used as a center of the search. This procedure is called multi-resolution Iterative Refinement Algorithm (IRA). Finally at full resolution, the GA based registration is performed on windowed images from the reference and input images. The algorithm reports the registration results in terms of rotation and x-y axis translation.

Recently, new multi-resolution approaches have been developed in image processing like the contourlet transform and its shift invariant transform, namely the nonsubsampling contourlet transform (NSCT). Serief *et al.* have proposed a new technique using the NSCT transform to primarily extract feature points for image registration (*Serief and al., 2009*). In this paper, we have oriented our work to apply the NSCT multi-resolution decomposed images in order to reduce the search space in combination with GAs for the purpose of search optimization.

### **V.3 An NSCT Enhancement Method for Image Registration**

Image enhancement is widely used in medical and biological imaging to improve the image quality. The purpose of image enhancement is to enhance weak edges or weak features in an image while keeping strong edges or features. Traditional image enhancement includes methods such as unsharp masking, split an image into different frequency subbands and amplify the highpass subbands. More recent methods are based on the discrete wavelet transform in a multiscale framework and achieve better results (*Laine and al., 1995*), (*Dippel and al., 2002*).



Owing to the geometric information, the contourlet transform achieves better results than discrete wavelet transform in image analysis applications such as denoising and texture retrieval (*Po & DO, 2006*). Due to downsampling and upsampling, the contourlet transform is shift-variant. However, shift-invariance is desirable in image analysis applications such as edge detection, contour characterization, and image enhancement (*Zhou and al., 2005*).

In this first part of the application of the NSCT to image registration, we present a method for image enhancement that is described as bellow.

### ***V.3.1 NSCT Image Enhancement Algorithm***

The NSCT provides not only multi-resolution analysis, but also geometric and directional representation. The NSCT is shift-invariant such that each pixel of the transform subbands corresponds to that of the original image in the same spatial location. Therefore, we gather the geometrical information pixel by pixel from the NSCT coefficients. In the frequency domain, both weak edges and noise lead to low-value coefficients. Since weak edges are geometric structures, while noises are not, we can use this geometric representation to distinguish them.

Based on this observation, we can classify pixels into three categories by analyzing the distribution of their coefficients in different subbands (*Zhou and al., 2005*). One simple way is to compute the mean (denoted by *mean*) and the maximum (denoted by *max*) magnitude of the coefficients for each pixel, and then classify it as shown in equation V.1.

$$\begin{cases} \text{strongedge} & \text{if } \text{mean} \geq c \sigma \\ \text{weak edge} & \text{if } \text{mean} < c \sigma, \text{max} \geq c \sigma \\ \text{noise} & \text{if } \text{mean} < c \sigma, \text{max} < c \sigma \end{cases} \quad (\text{V.1})$$

Where *c* is a parameter ranging from 1 to 5, and  $\sigma$  is the noise standard deviation of the subbands at a specific level.

The goal of image enhancement is to amplify weak edges and to suppress noises. To this end, we modify the NSCT coefficients according to the category of each pixel by a nonlinear mapping function

$$y(x) = \begin{cases} x & \text{strong edge pixel} \\ \max\left(\left(\frac{c\sigma}{|x|}\right)^p, 1\right)x & \text{weak edge pixels} \\ 0 & \text{noise pixels} \end{cases} \quad (\text{V.2})$$

Where the input  $x$  is the original coefficients, and  $0 < p < 1$  is the amplifying ratio.

The enhancement algorithm is summarized as follows (Zhou and al., 2005):

1. Compute the NSCT of the input image for  $N$  levels.
2. Estimate the noise standard deviation of the input image.
3. for each level DBF,
  - (a) Estimate the noise variance.
  - (b) Compute the threshold and the amplifying ratio.
  - (c) At each pixel, compute the mean and the maximum magnitude of all directional subbands at this level, and classify it by (2) into strong edges, weak edges, or noises.
  - (d) For each directional subband, use the nonlinear mapping function given in (3) to modify the NSCT coefficients.
4. Reconstruct the enhanced image from the modified NSCT coefficients.

### ***V.3.2 Simulation Results***

The parameters of GAs used in this test are: The population size in each generation is restricted to 50 individuals with a crossover probability of 0.8 and a mutation probability of 0.01. The GA meets the criterion within 100 generations. For modifying the NSCT coefficients, we choose  $c=4$  and  $p=0.3$

We have applied our algorithm of registration based on the NSCT enhancement method on SPOT satellite images (Meskine and al., 2009B). The tested pair of images to be compared used with the same initial parameters as the previous experiments.

In order to evaluate the effectiveness of this registration method, we have conducted a comparison between this approach and the multiresolution registration strategy developed in the previous chapter based on the wavelet decomposition. The accuracy of this search increases when going from coarse resolution to fine resolution. The corresponding sub-band couples from the two images are compared using GAs. The purpose is to maximize the correlation coefficient between the two images, in order to find the best parameters of transformation (R,X,Y). The same procedure is applied with the NSCT decomposition alone and with the enhancement method.

Two measures are considered to determine the accuracy of registration: the correlation coefficient and the root mean square error defined as:

$$RMSE = \sqrt{\frac{1}{k} \sum_{i=1}^k (P_i - Q_i)^2} \quad (V.3)$$

Where  $P$  is the original image and  $Q$  the corrected one, and  $k$  is the size of the image.

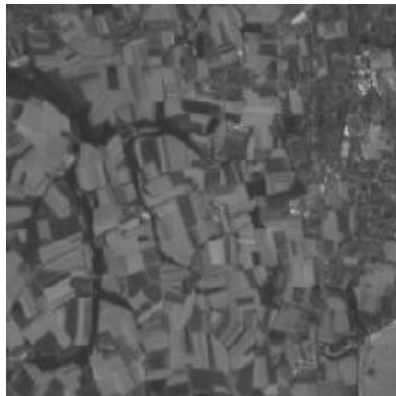
Table V.1 depicts the analytical results obtained for these methods using the correlation coefficient and the RMSE values.

Method	RMSE	Corr.Coeff
Wavelet	0.2583	<b>0.7781</b>
NSCT	0.1634	<b>0.9136</b>
NSCT+Classif.	<b>0.1269</b>	<b>0.9419</b>

**TABLE V.1.** Comparison of RMSE and correlation coefficient Values obtained using different methods.

Figure V.1 shows the two images to be compared of size (256\*256) pixels and the image registration results using the three applied transforms; wavelet, NSCT and NSCT with classification (enhancement method).

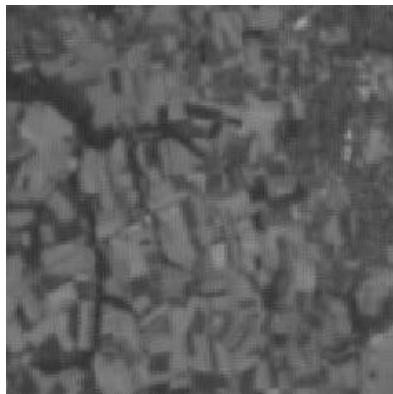
It is clear from the analytical results and the quality of these images is best obtained using the NSCT enhancement method.



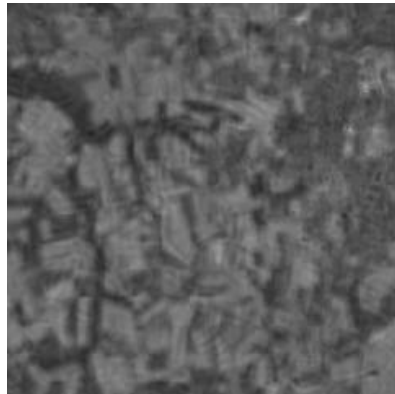
(a)



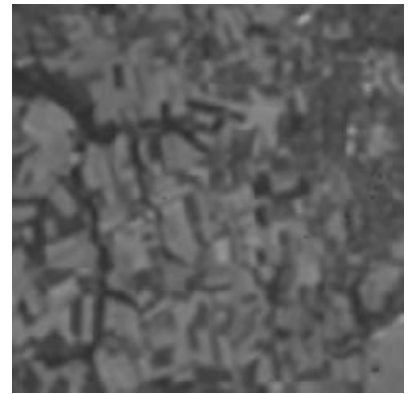
(b)



(c)



(d)



(e)

**Figure V.1.** The resulting of SPOT image registration obtained using different transforms: (a) Reference image, (b) transformed image to be compared, (c) registered image using wavelet decomposition, (d) registered image using NSCT method, and (e) the registered image using NSCT with classification.

#### **V.4 Proposed Methods of Registration**

In this work, a better NSCT-GA approach for image registration is being proposed. We have utilized two techniques to accomplish registration multi-resolution decomposed images in order to reduce the search space and GAs for optimization of the search space (*Meskine and al., 2010*). The multi-resolution decomposed image employs two transforms: wavelets and NSCT.

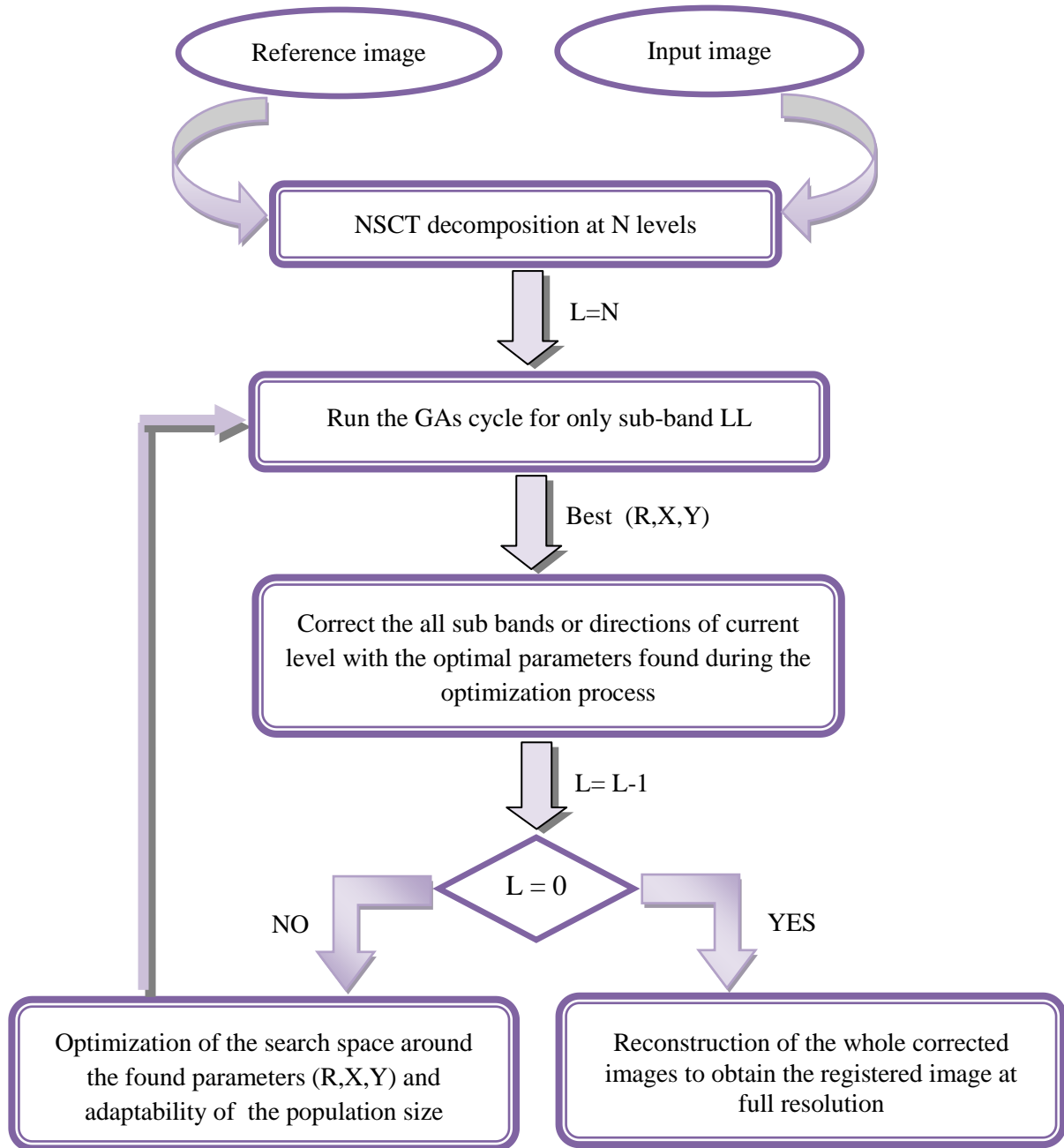
While using the wavelet transform, both the reference image and input or transformed image to be corrected are first decomposed following multi-resolution wavelet decomposition strategy. At each level of the decomposition, the search focuses in the interval around the “best” transformation found at the previous level and is refined at the next level up; working iteratively from the deepest level of decomposition (where the image size is the smallest) to the top level of decomposition, i.e going from coarse to fine spatial resolution. In other words, the parameters found in level  $l$  are used to estimate the new search space of GAs of level  $l-1$  with minimizing the population size in order to reduce the search space. The accuracy of this search increases when going from coarse resolution to fine resolution. The registration process terminates when the matching criteria is optimized at the highest resolution level. At fine resolution, we reconstruct the corrected image.

The second multi-resolution decomposition used in this work is the NSCT in which we propose two methods of registration based on this transform. In both ones, we proceed as in the wavelet transform but the NSCT decomposition has more directions or sub-bands.

In the first proposed method, we decompose the two images to be compared into several levels with different directions. The decomposition results are one LL sub-band, which is the approximation of the original image, and different direction sub-bands. At each level of the decomposition, the correlation ratio between corresponding sub-band images of the reference and input images is successively computed and maximized using GAs. We correct the sub-bands of the input image with the optimal transformation parameters (R,X,Y) found during the run process at this level. These obtained parameters are used to refine the search space of the lower level.

Going from one level to another is done according to two criteria: concentrate the search space around the optimal values found in the previous level, then adapt better the population size by minimising it. This way, the time complexity of the refinement process through different levels is really reduced, while the registration accuracy is increased. This is indeed an adaptable GA. Finally, the corrected image is reconstructed at full resolution with all different corrected sub-bands.

The results obtained by the first proposed method are better than those of the one based on the wavelet, although the size of the different sub-bands of all levels is the same as that of the original image in the NSCT decomposition because it is a shift-invariant multi-resolution transform; as opposed to the wavelet transform, in which the size of sub-bands decreases with the increase of the level. To overcome this problem, we propose a second method based on the NSCT. This approach consists of performing the GA's process on only the sub-band LL. At each level of the decomposition, the parameters found with GAs are used to correct the directions or sub-bands of this level. The level changing process is done the same way as that of the dynamic and adaptable one in the first method. At full resolution, we reconstruct the registered image with the corrected sub-band images of all directions. This algorithm is shown in Fig.V.2.



**Figure V.2.** Image registration algorithm flowchart based on the second proposed NSCT method.

## **V.5 Results & Discussion of the proposed method**

The GA parameters used in this work are set as follows: the population size in each generation is initially restricted to 80 individuals with a crossover probability of 0.85 and a mutation probability of 0.02, and the algorithm fulfils its generations at a maximum of 100 iterations of processing.

In this work, for the different level decompositions, the Haar filter has been used for the wavelet transform, while the diamond maxflat filters have been used for the NSCT transform for both directional and pyramidal filters. The wavelet decomposition is carried out up to the third level.

As for the NSCT decomposition, we have chosen two levels with four directions each, for both methods. Initially, a population size of 80 has been set for the highest level. When switching from one level to a lower one, this size is reduced by a step of 30 individuals. Thus, at the lowest level, this size will be of only 20 individuals. Moreover, the search space is reduced at each level and is concentrated around the optimal parameter values found in the previous level.

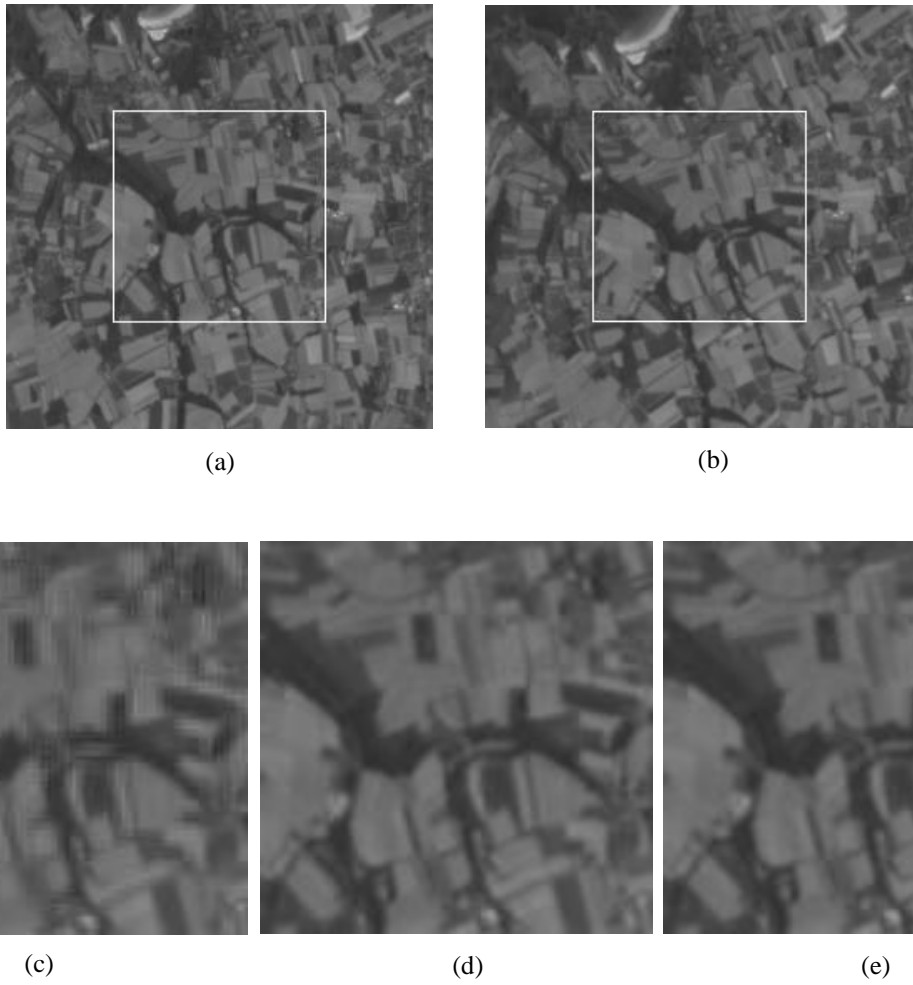
The experimental tests were performed using different types of satellite images such as SPOT and IKONOS as well as radar images. In these first experiments, the transformed or input images to be corrected are simply the reference images rotated by a 7 degrees angle of rotation and displaced by a (13×9) pixel translation in the X and Y directions from the center of the reference images. The sensed images are resampled using the bilinear interpolation.

Figure V.3 illustrates a SPOT registered image obtained using the three multi-resolution approaches: wavelet, and the two NSCT methods. The pair of images to be compared is shown with a size of (256×256) pixels, in which the region of interest is enclosed in the white box. Thus, the resulting image is of a size of (128×128) pixels.

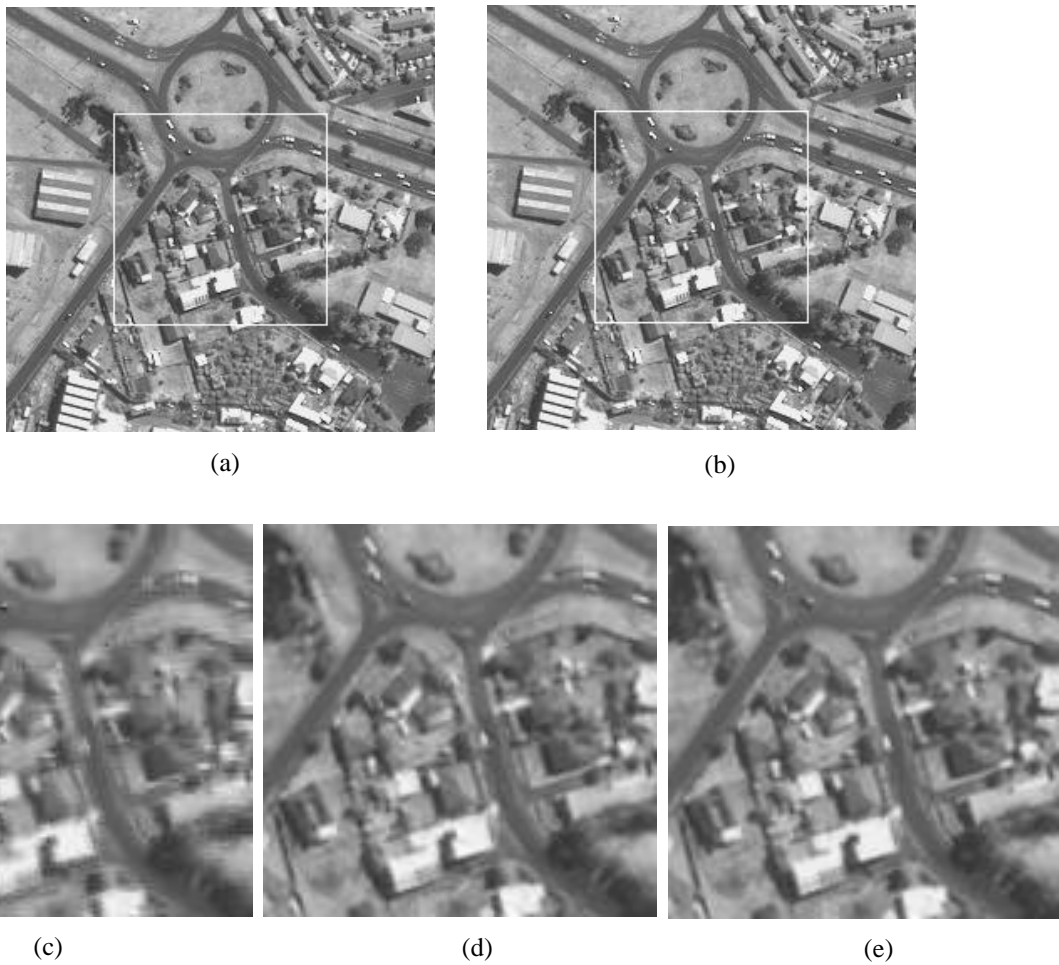
We have also tested our registration algorithm on an IKONOS image and a SAR image (with inherent speckle noise), as shown in Fig.V.4 and V.5, respectively.

Moreover, the different analytical results of the registration methods on all image sets are depicted in Tables V.2. In addition, the computation cost when using an HP Compaq machine, core 2 duo 2.66 GHz CPU is also considered.

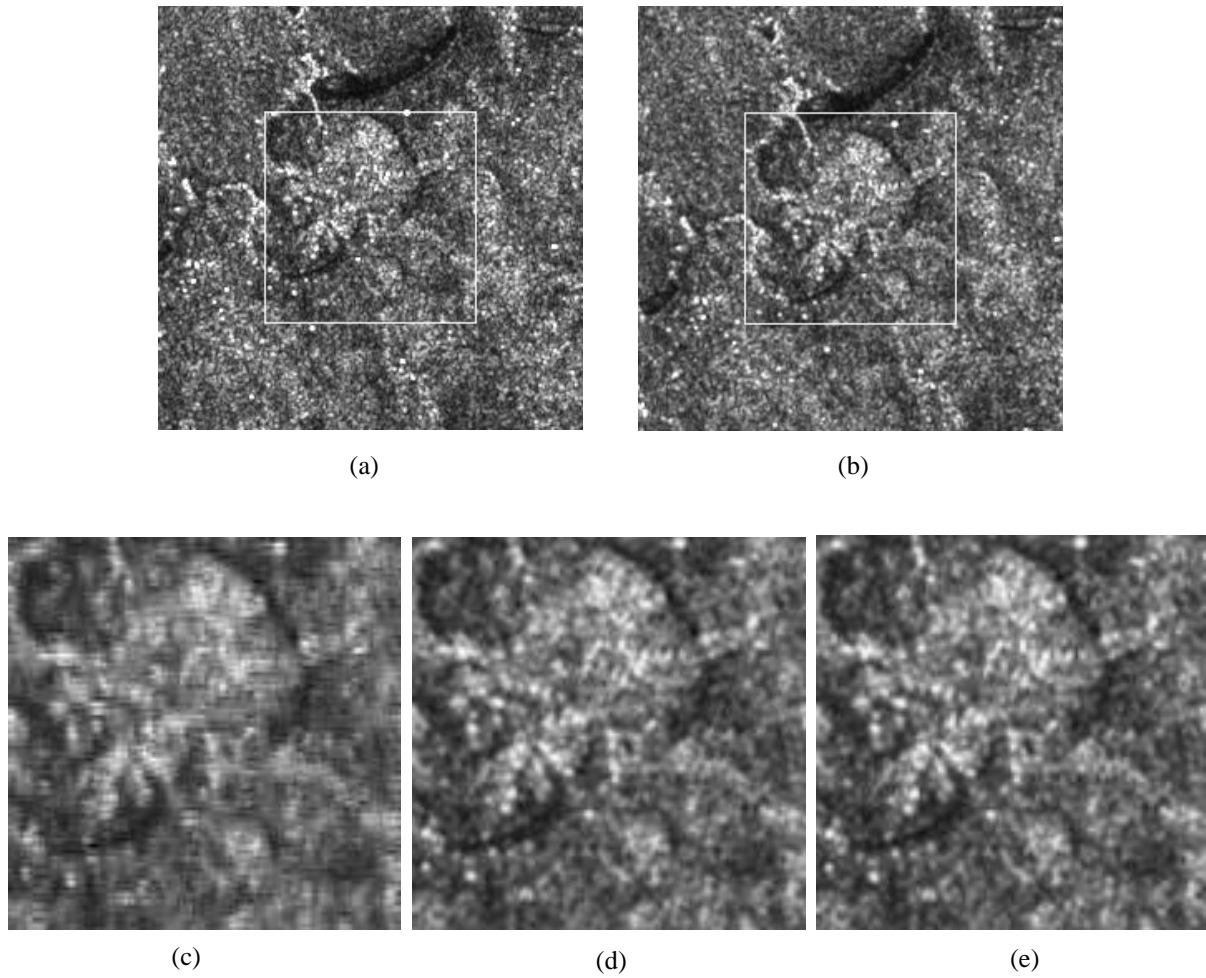




**Figure V.3.** Registration of SPOT images using the three methods: (a) reference image, (b) transformed image to be corrected, (c) registered image using wavelet, (d) registered image using the first proposed method, and (e) the corrected image using the second proposed method.



**Figure V.4.** Registration of IKONOS images obtained using the three approaches: (a) the reference image, (b) the transformed image, (c) registered image using wavelet, (d) registered image using the first proposed method, and (e) registered image using the second proposed method.



**Figure V.5.** Registration of SAR images (with inherent speckle noise) using the three methods: (a) reference image, (b) transformed image to be corrected, (c) registered image using wavelet, (d) registered image using the first proposed method, and (e) the corrected image using the second proposed method.

		Techniques		
		Wavelet	NSCT1	NSCT2
SPOT	Corr.ratio	0.8298	0.9863	0.9840
	RMSE	0.0462	0.0137	0.0147
	Computation time (s)	929.25	5127.04	1421.20
IKONOS	Corr.ratio	0.6935	0.9562	0.9468
	RMSE	0.1245	0.0504	0.0549
	Computation time (s)	1114.73	5213.72	1447.58
SAR	Corr.ratio	0.5622	0.9474	0.9294
	RMSE	0.1549	0.0608	0.0703
	Computation time (s)	1065.54	5206.86	1448.27

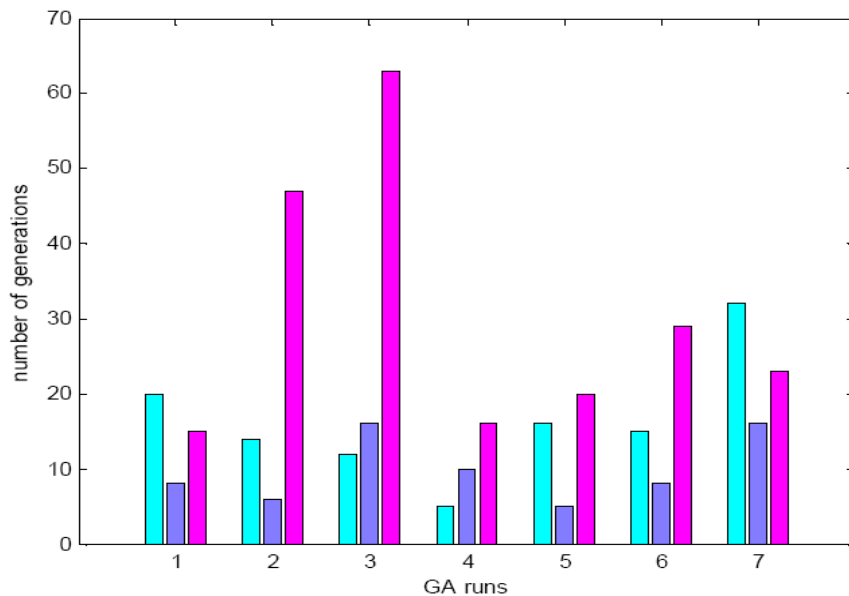
**Table V.2.** Analytical results for the three registration methods applied to all images.

It is clear from these results that both the NSCT proposed methods perform better in terms of correlation and RMSE than the wavelet method. It is noteworthy that the processing time is lower when performing the NSCT2 method than that of the NSCT1 one. This is due to the fact that we used all sub bands for the first method while we used only the LL sub band for the second one. Moreover, the NSCT2 has been compared to the regular registration method and the results in Table V.3 show a significant time processing reduction when using NSCT2.

Methods	Coor.Ratio	RMSE	Computation time (s)
Regular	0.9845	0.0144	167220.00
NSCT2	0.9840	0.0147	<b>1421.20</b>

**Table V.3.** Comparison results between the NSCT2 and regular methods.

Although, it is well known that GAs are essentially stochastic search and optimization methods. Results obtained by GAs are only meaningful on a statistical basis since different runs of a GA may lead to different optimal solutions. For a very simple problem, the final optimal solutions obtained may be the same for different runs, but the numbers of generations at which the optimal solutions are obtained could be different. Figure V.6 presents the statistical results of the NSCT2 proposed method for an example, in which seven GAs runs have been performed on a case of SPOT images to show the behaviour of these techniques. Two levels of decomposition are used which have lead to three LL sub-bands. For each GA run, the number of generations for levels 0, 1 and 2 are shown from left to right, respectively.

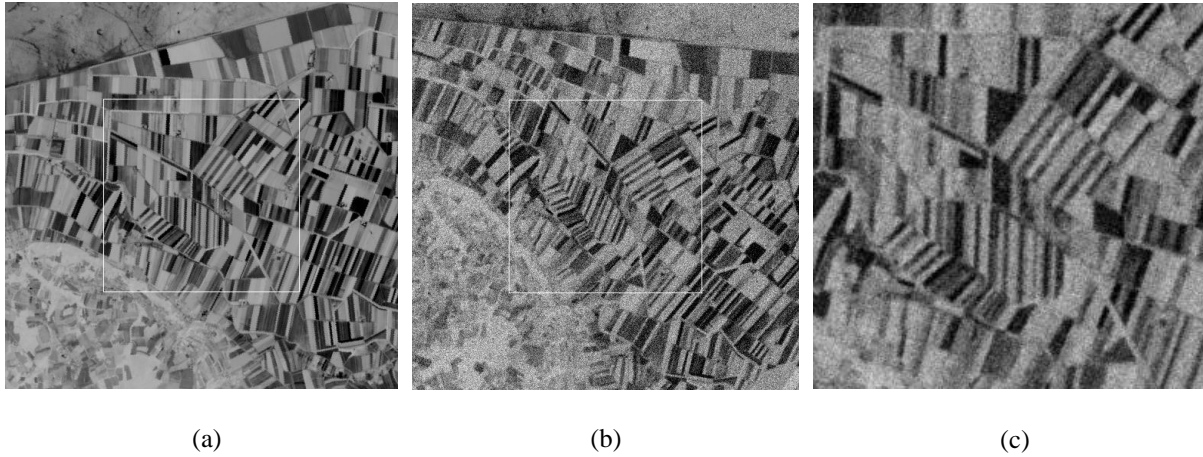


**Figure V.6.** Statistical results for an NSCT2 application.

### **Real Applications Results**

The proposed algorithm works perfectly well in real practical applications as well, even with the presence of noise as shown in Fig.V.7, which presents the registration results of multi-temporal satellite images (*Meskine and al., 2010*). The tested pair of images is one of panchromatic SPOT images acquired at different dates, on one of which we have added a Gaussian white noise.

Clearly, the obtained results are really promising. The analytical results obtained with the second proposed method led to an RMSE of 0.0630 and a correlation ratio of about 0.9317.



**Figure V.7.** Registration of multi-temporal SPOT images: (a) the input image, (b) the sensed noisy image to be corrected, and (c) the registered image with the second proposed method.

## V.6 CONCLUSION

In this chapter, we have described our main contributions in the field of image registration. An efficient algorithm of image registration is presented that uses GAs within a multi-resolution framework based on the NSCT which provides a directional multi-resolution image representation for performing an efficient, robust and accurate rigid registration. An adaptable GA is adopted to speed up the search of the best transformation parameters. A comparative study between wavelets and the two proposed approaches of the NSCT registration is conducted. Experimental results show that the NSCT is a promising method for registration of satellite images compared to both the wavelet method and the NSCT enhancement image registration method.



# CHAPTER VII

## ***A 2D POINT- BASED REGISTRATION TECHNIQUE USING GAS***

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## VI.1 Introduction

2D/3D image registration and mapping is a key problem in computer vision that shows up in a wide variety of applications such as medical image analysis, object tracking, recognition and visualization.

The process of image Registration consists of finding the spatial alignment between two images. Fitzpatrick defines it as “the determination of a geometrical transformation that aligns points in one view of an object with corresponding points in another view” (Fitzpatrick and al., 1998). The term view is used to include images as well as real 3D objects. Zitova emphasizes that the views to be registered could have been taken at different times, from different view points, using different equipment (multi-modal), or could even be on different subjects (inter-patient) (Zitova & Flusser, 2003).

It can be formulated as a problem of optimizing a function that quantifies the match between the original and the transformed image. Several image features have been used for the matching process, depending on the modalities used, the specific application and the implementation of the transformation. The registration process can be divided into three main categories: point-based, surface-based and volume-based methods.

*Point-based* registration involves the determination of the co-ordinates of corresponding points in different images and the estimation of geometrical transformation using these corresponding points. Then, the task of registration is to place the data into a common reference frame by estimating the transformations between the datasets. What makes the problem difficult is that correspondences between the point sets are unknown a-priori. A popular approach to solving the problem is the class of algorithms based on the Iterated Closest Point.

The ICP (originally Iterative Closest Point) algorithm described by Besl and McKay (Besl & MacKay, 1992) is well known for aligning 3D object models. Originally ICP starts with two data sets (mostly points) and an initial guess for their rigid body motion. Then the transformation is refined by repeatedly generating pairs of corresponding points of the sets and minimizing an error metric. ICP algorithms are mostly applied to 2D or 3D point sets (Stoddart & Hilton, 1996).



ICP is attractive because of its simplicity and its performance. Although the initial estimate does need to be reasonably good, the algorithm converges relatively quickly to a local minimum. Many variants of ICP have been proposed to address these limitations. Chen and Medioni developed a mechanism which minimizes the summation of the square distances between points on a view in reference to a tangent plane on another view (*Chen & Medioni, 1992*). Some comparative studies of ICP variants have been made in (*Rusinkiewicz & Levoy, 2001*) and (*Dalley & Flynn, 2002*). A highly detailed survey on the registration methods as well as recognition and 3D modeling techniques was given in (*Campbell & Flynn, 2001*).

The application of several well-known evolutionary algorithms to the image registration optimization process has introduced an outstanding interest in order to solve those problems due to their global optimization techniques nature. In the broad area of global optimization methods, GAs form a widely accepted trade-off between global and local search strategies. They are well-investigated and have proven their applicability in many fields. A number of authors have used GAs for full-view image matching in various forms. Renner and Ek'art provide a summary of genetic algorithms in computer aided design (*Gàbor & Anik'ó, 2003*). (*Jacq & Roux, 1995*) use GAs for registration of 3D medical images. (*Brunnström & Stoddart, 1996*) used a GA to find an initial guess for the free-form matching problem that is finding the translation and the rotation between an object and a model surface. GAs have also been applied to a range image registration problems as works of (*Yamany and al., 1998*), (*Robertson & Fisher, 2002*), and (*Silva and al.2003*) which used the GAs to estimate the transformation parameters in the ICP algorithm. They showed that their algorithm can finally converge to the optimum registration parameters but it may take a long time. In contrast to the 2D–3D registration, numerous methods exist to precisely register 3D data by iterative algorithms like the Iterative Closest Point and its variants (*Dmitry and al.2005*), (*Chetverikov and al., 2005*), (*Chow and al., 2004*) and evolutionary algorithms (*Cordón and al., 2003*).

In this chapter, we present an efficient 2D point based rigid image registration method integrating the advantage of the robustness of GAs in finding the best transformation between two images. The algorithm is applied for registering SPOT images as well as medical IRM images. The high-quality results show that the proposed approach is highly-automatic and reliable.

## V.2 ICP Algorithm

The ICP algorithm has become the dominant method for aligning three dimensional models based purely on the geometry, and sometimes color, of the meshes. It has been widely used in a variety of fields such as medical images (McLaughlin and al., 2005), fingerprint images (Jain ad al., 2007) etc. Different works concerning ICP algorithms can be found in (Czopf and al. 1999), (Huber & Hebert., 2001).

Since his introduction in 1992, there have been many variations to mitigate its deficiencies. This algorithm formulated a basic scheme to obtain the alignment while minimizing the cost function and is based on the squares summation of the distance between points on the image. The basic procedure involves features identification, matching of correspondent features, and the alignment of these matches, by evaluating a distance. It is one of the most efficient algorithms for robust rigid registration of 3D data. Actually, ICP was proposed to solve the curve matching problem as well in both 2D and 3D space (Zang, 1994). It consists in finding the closest points between two sets of data which are then used to estimate the parameters of the global rigid transformation to register the two data sets. Figure VI.1 shows the concept of ICP algorithm.

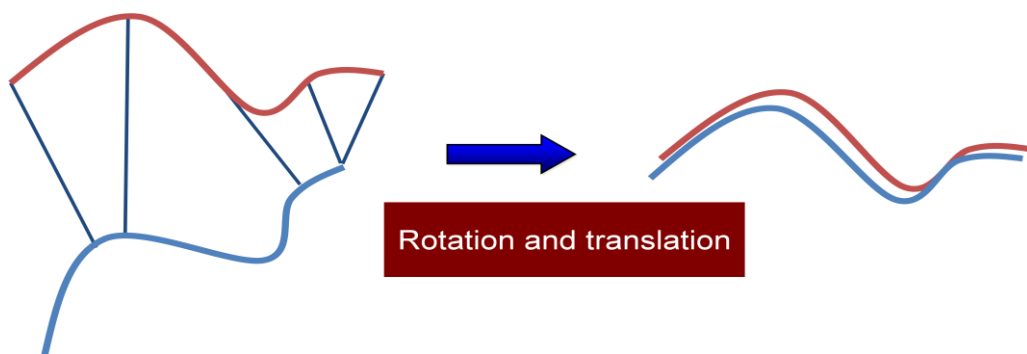


Figure VI.1. ICP algorithm.

The ICP algorithm implements a natural idea: given motion parameters, for all points in the first image, the closest points in the second image to the transformed points must represent their correspondents. This idea is so practical and effective that it has attracted much attention from the machine vision and pattern recognition community (Yonghuai, 2004).

## OVERVIEW OF THE ICP ALGORITHM

The ICP algorithm is perhaps the most widely used technique in point matching. The method iteratively solves for the correspondence and transformation, and it can be applied to both affine and rigid registrations. The correspondence is determined with the nearest neighbor criteria once the transformation is fixed, and the transformation is determined by solving a linear system of equations when the correspondence is fixed.

Let us assume that the given two surfaces to be registered can be described as point sets; the scene data points,  $\mathbf{P}$ , with  $N_p$  points,  $\{p_i, i=1, \dots, N_p\}$ , and the reference data points,  $\mathbf{M}$ , with  $N_m$  points,  $\{m_j, j=1, \dots, N_m\}$ . Depending upon the sampling of the surfaces,  $N_p$  is not necessarily equal to  $N_m$ . Furthermore, the point  $p_i$  of the scene surface does not necessarily represent an exact 3D correspondence to the point  $m_j$  of the reference surface. However, the search space is determined by the size of the scene data set; i.e.,  $N_p$ .

The ICP algorithm can be summarized as follows (*Almhdie and al., 2007*):

### A. Initialization:

- 1) Let the initial scene surface  $\mathbf{P}_0$ , be equal to  $\mathbf{P}$ .
- 2) Define the maximum number of iterations  $k_{\max}$ .
- 3) Initialize the translation vector and the rotation matrix as follows:

$$T = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

with the initial coefficient of the translation vector and rotation matrix set as follows:  $t_u = 0, r_{uv} = 0$  if  $u \neq v$ , and  $r_{uu} = 1, u = 1, 2, 3, v = 1, 2, 3$ . This corresponds to zero translation and no rotation.

**B. Iterations:**

1) For each point  $p_i$  ( $i=1, \dots, N_p$ ) of the scene  $\mathbf{P}$ , compute the closest point  $m_j \in \mathbf{M}$  from the model using the Euclidian distance. Let  $\hat{m}_i$  be the point on  $\mathbf{M}$  corresponding to the minimum distance to  $p_i$ .

2) Using the selected correspondence pairs, compute the transformation, rotation ( $\mathbf{R}$ ) and translation ( $\mathbf{T}$ ) that minimizes the mean square error (MSE) of the estimated correspondence pairs:

$$MSE = \frac{1}{N_p} \sum_{i=1}^{N_p} \|\hat{m}_i - R(p_i) - T\|^2 \quad (VI.1)$$

Different close-form solution techniques of the original ICP algorithm can be used, i.e., quaternion (*Horn, 1987*), (*Mukundan, 2002*) or single value decomposition (*Arun and al., 1987*).

The resulting transformation from the minimization of the above equation at step  $k$  will be denoted  $\mathbf{R}_k$  and  $\mathbf{T}_k$ . This step also provides the minimum distances which correspond to the matched pairs.

3) Compute  $\mathbf{P} = \mathbf{R}_k \times \mathbf{P}_0 + \mathbf{T}_k$  and restart a new iteration if the change in the MSE is above a predefined threshold  $\zeta$ , and if the maximum number of iterations  $\mathbf{k}_{max}$  is not reached. If not, stop the iterations and exit.

The main drawback of the ICP algorithm is its sensitivity to the initial alignment. If the two surfaces are not initially closely aligned, then the closest points between the two surfaces might result in false matches. Subsequently, the false matches may cause the ICP algorithm to calculate an inaccurate transformation which results in failure of the ICP algorithm to find the optimal solution (*Almhdie and al., 2007*).

### IV.3 Registration Methodology

The registration process is usually carried out in four steps. The first step consists of the selection of features on the images. Next, each feature in one image is compared with potential corresponding features in the other one. A pair of points with similar attributes is accepted as matches and they are called control points. Finally the parameters of the best transformation which models the deformation between both images are estimated using the control points obtained in the previous step (*Bentoutou and al. 2005*).

In order to get reasonably good registration results, an operator has to choose a considerably large number of feature pairs across the whole images, which is not only tedious and wearing but also subject to inconsistency and limited accuracy. Thus, there is a natural need to develop automated techniques that require little or no operator supervision. In this part of the work, we propose a point registration method based on GAs applied to satellite images. This procedure is focused on the problem of obtaining the best solution between two data point sets generated from both images to be compared through a distance matching; and which is compared with the previous proposed methods.

The formulation that we shall consider is based on the following characteristics:

**Input space:** the images are assumed to be represented as a discrete set of 2D feature points. In our experiments, extracted feature points algorithm is based on the NSCT decomposition.

**Search space:** our software system support rigid transformations with three considered parameters; rotation and translation in both x and y directions. The user provides an interval limiting the range for each parameter.

**Search strategy:** our algorithm is based on searching the transformation space for the optimal aligning transformation. It employs genetic algorithms as a global optimization method to find the optimal transformation.

**Similarity metric:** for our application, the similarity measure is based on the Euclidean distance. The objective is to minimize the error distance between the corresponding pair of data sets.

### VI.3.1 NSCT-based Feature Points Extraction Method

The purpose of the feature extraction is to derive features that describe image characteristics that are relevant in a co-registration process and which can be used to select a subset of regions and choose an appropriate method for each. The feature extraction approach used in this paper exploits a nonsubsampling directional multi-resolution image representation to capture significant image features across spatial and directional resolutions.

The proposed feature extraction method is described in the following algorithm (Serief and al., 2009):

**Step 1:** Compute the NSCT coefficients of both images for N levels and L directional subbands.

**Step 2:** Compute the difference between each directional subband at one level and the corresponding one at another level. L difference subbands will be obtained at the end.

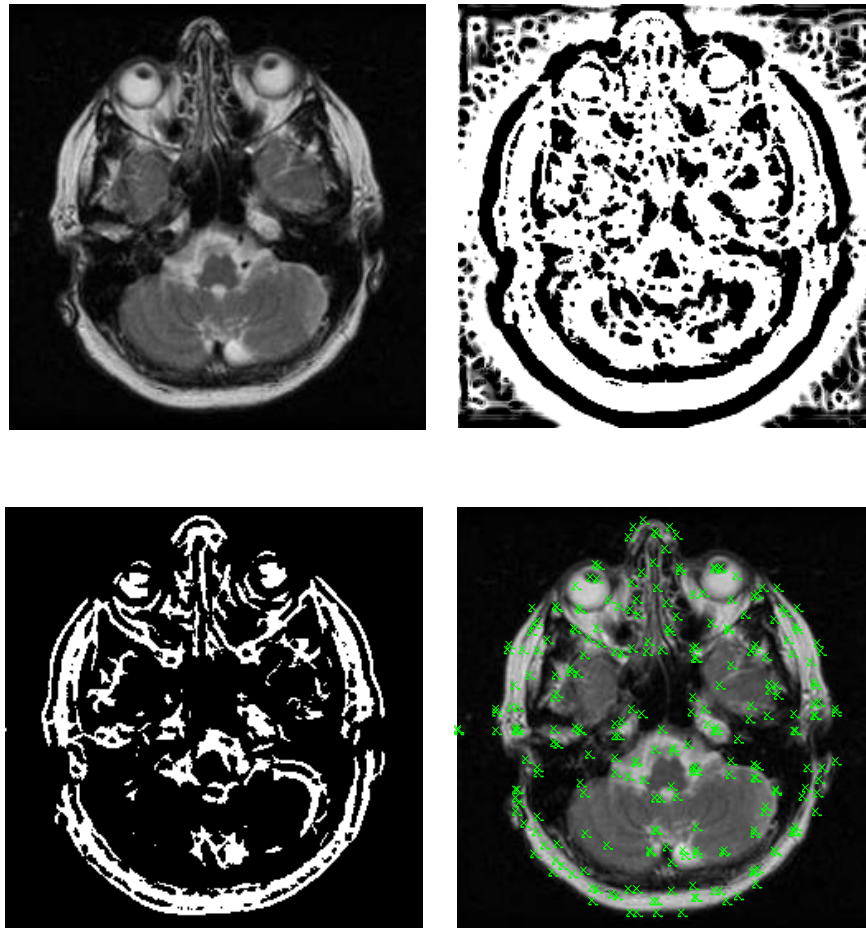
**Step 3:** At each pixel location, compute the maximum magnitude of all obtained difference subbands. These points are called “maxima of the NSCT coefficients”.

**Step 4:** A hard thresholding procedure is then applied on the NSCT maxima image in order to eliminate non significant feature points. A point is recorded if NSCT maxima  $>$  Th,

Where  $Th = c(\sigma + \mu)$ , c is a parameter whose value is defined by the user, and  $\sigma$  and  $\mu$  are the standard deviation and mean of the NSCT maxima image, respectively.

**Step 5:** Take a block neighborhood of size  $w \times w$  and find one local maximum in each neighbourhood, this will eliminate maxima that are very close to each other. The locations of the obtained thresholded NSCT maxima are taken as the extracted feature points.

An example of feature point extraction is shown in the following figure for IRM image.



**Figure VI.2.** Feature point extraction: in the top left the original image and in the right the NSCT maxima. At the bottom of the figure: in the left the thresholded NSCT maxima, and in the right the resulting control points selected.

**CP candidates matching**

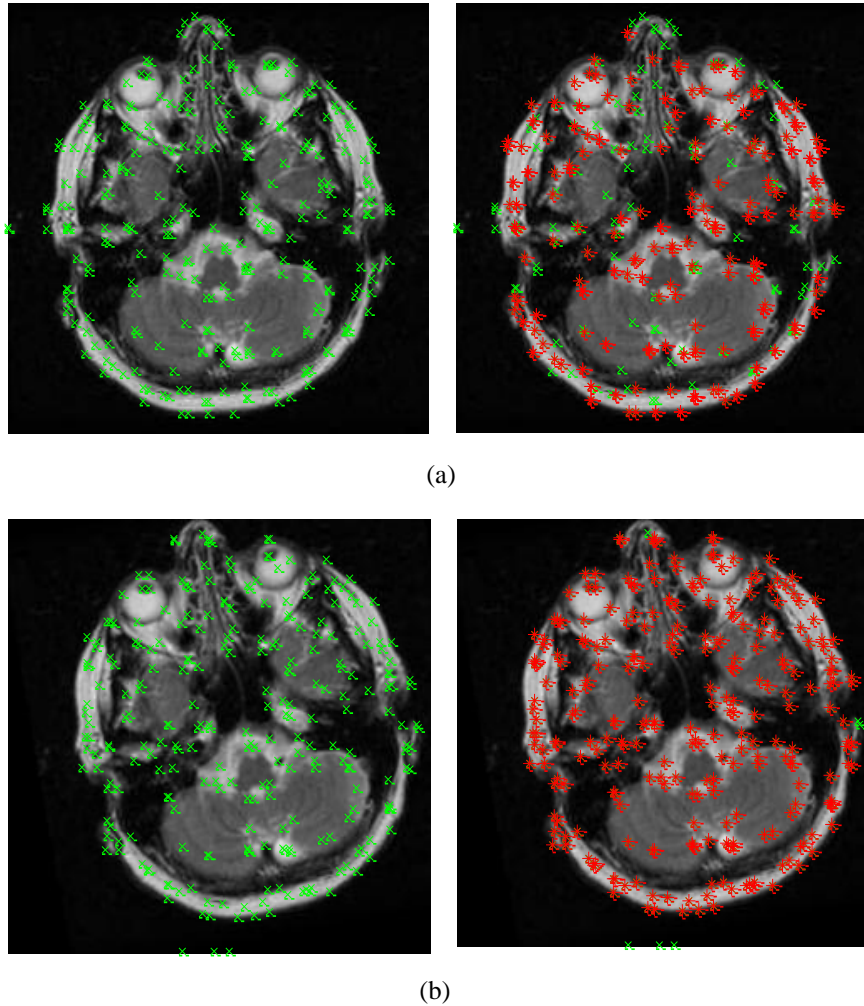
After the feature points are detected from the images to be registered, a correspondence mechanism between these two feature points sets must be established in order to refine the control points. The objective is that each feature point in the reference image is paired with its correspondent in the sensed image. In this work, the correlation based similarity measure is used to establish the correspondence between the two feature point sets.

The algorithm of points matching is abstracted in three main steps:

1. For every control points, choose a moving circular template of radius  $\rho$  centered at this point in the reference image.
2. Calculate the correlation coefficient of the window template and the corresponding template  $w_2$  centered at  $q_j$  in the sensed image.
3. The control point  $q_i \in Q$  that might correspond to the candidate point  $p_i \in P$  in the reference image is determined by the shift giving the maximum correlation.

An example of the control points matching is illustrated in Fig.VI.3.





**Figure VI.3.** Corresponding points identification between two images: (a) interest points of the first image and (b) the interest points of the second image. At the left for both images, the extracted points with NSCT method before the matching process marked with green cross; and in the right the interest points identified after the matching marked with red cross.

### IV.3.2 Similarity Metric

After a preprocessing of the data, the registration proceeds as an optimization of the function which measures the match over the selected parameters of the geometric transformation.

A GA uses a fitness function to determine the performance of each artificially created chromosome; therefore the fitness function should measure the registration quality of each chromosome. A GA should try to find a chromosome with the minimum Euclidean distance between each correspondence pair (*Chow and al., 2002*).

Formally, given two finite size point sets, the *model* set  $M$  and the *scene* set  $S$ , our registration method finds the parameters of a transformation  $T$  which minimizes the cost function. Then, in this work, we adopt this modified distance as an objective function of GAs which will be minimized. We assume that the type of transformation is rigid with three parameters: rotation and translation in both x and y directions.

For the data point in the model image with coordinates  $x$ ,  $y$  and intensity value  $I$ , its image is  $x'$ ,  $y'$ , and  $I'$ , and are related by the mapping given in the following equation.

$$Pt = R * p_i + T \quad (VI.2)$$

Where,

$p_i(x, y)$  is a source point

$Pt(x', y')$  its transformed corresponding point

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is a rotation matrix}$$

$$T = \begin{bmatrix} Tx \\ Ty \\ 0 \end{bmatrix} \text{ is a translation vector.}$$

Assume that the given two data sets to be matched are  $P = \{p_1, p_2, \dots, p_m\}$  and  $Q = \{q_1, q_2, \dots, q_n\}$  where  $m$  is not necessarily equal to  $n$ .

If the registration parameters are given, then for any point  $p_i$  in  $P$ , we can use the following criterion to determine its possible correspondent  $q_i$  in  $Q$ :

$$q_i = \operatorname{argmin} \|q - (Rp_i + T)\| \quad (\text{VI.3})$$

Thus the objective is to minimize the Euclidean distance between the transformed point  $Rp_i + T$  and  $q$  in the  $Q$ . A suitable transform means the distance error between  $P$  and  $Q$  is minimized. Therefore, the *fitness function* of GA to be *minimized* is described in the following equation

$$F = \operatorname{median} \|p_i - q_i\|. \quad (\text{VI.4})$$

The evaluation of the fitness function described above requires a search on the closest point from a data set given an input data point.

In relation to image registration, every individual represents a combination of all transformation parameters which describes an image transformation. Therefore, the vector of parameters (R, X, Y) is used as a chromosome that participates in an iterative process. All representations are signed magnitude, using one bit for the sign and the rest of the bits to represent the magnitude of the rotation or translation. Thus, the relative rotation has a range of  $\pm 128$  degrees, while the relative translation in the x (or y) direction has a range of  $\pm 32$  pixels. Thus, altogether, they could represent as many as  $2^{20}$  different elements by this string.

## **VI.4 Results of Application to Satellite Images**

The parameters of GAs used in this test are: the population size in each generation is restricted to 100 individuals with a crossover probability of 0.75 and a mutation probability of 0.05. The GA meets the criterion within 200 generations. To improve the performance of GAs, we have used two techniques named elitism and fitness sharing.

As previously, we have applied our proposed algorithm of registration on SPOT satellite images (*Meskine and al., 2012*).

### ***VI.4.1 First Experiment***

Evaluation of the fitness function described above requires a search on the closest point from a data set given an input data point. The corresponding searching time will be very long and becomes a major obstacle in utilizing the GA approach for practical applications. Therefore, in the first experiment and in order to limit the point set representing the image, we choose a window of size 40\*40 pixels from the center of both images in order to have about 1600 points at each model.

Figures VI.4 and VI.5 illustrate the performance of the GA process during the run. In Fig.VI.4, we see the evolution of the best fitness value at each generation. This value which is median (dist) is minimized from generation to another until the optimal fitness value is found and which corresponds to the optimal solution. Figure VI.5 depicts the evolution of the parameters (R,X,Y) during the generations. The red dashed lines show the initial parameters and blue lines show the optimal parameters found during the run the GA. We see that the optimal parameters values are closer to the initial parameters.

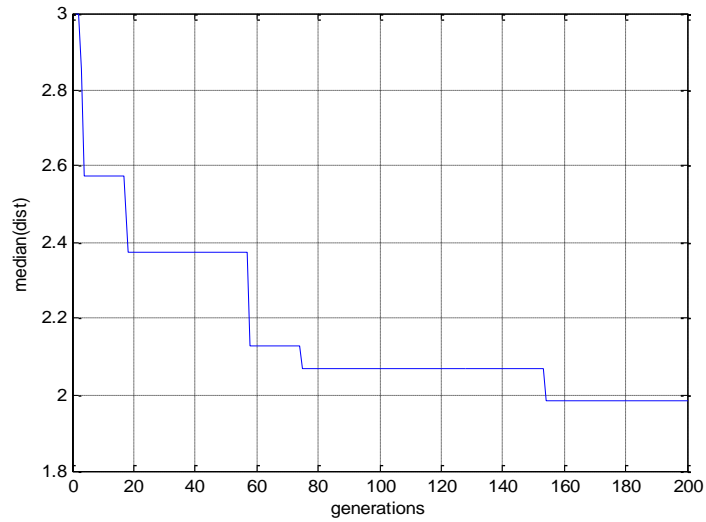


Figure VI.4. Evolution of the best fitness during the GA run.

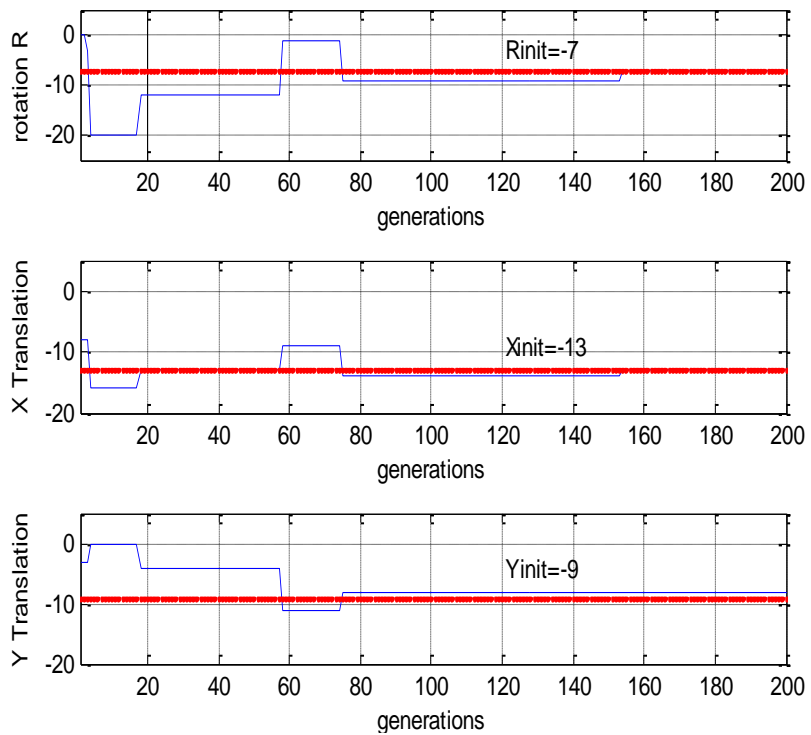


Figure VI.5. Evolution of the transformation parameters during the GA run.

The performance of the proposed registration method is also evaluated against other registration methods. The results of the parameters transformation found with this technique of GA based point registration noted by ‘GA proposed’ is compared with the ICP algorithm (noted ICP) and the intensity registration based method (noted GA intensity) for which the objective is to maximize the correlation coefficient of the two images.

The analytical results are depicted in table VI.1. The results found with ‘GA proposed’ are similar to those of ICP. However, the results of the GA intensity method are slightly different particularly for the X translation. So, we can say that the point registration is more robust and accurate than those of intensity methods.

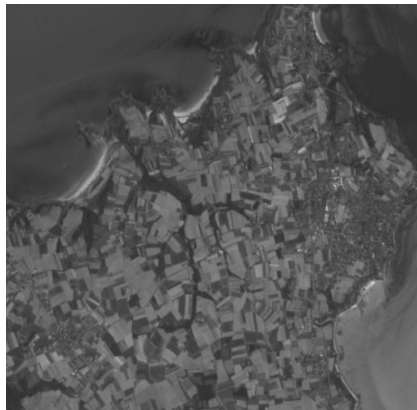
Methods	Rotation R	Translation X	Translation Y
<i>ICP</i>	<b>7</b>	<b>13</b>	<b>8</b>
<i>GA intensity</i>	<b>7</b>	<b>10</b>	<b>8</b>
<i>GA proposed</i>	<b>7</b>	<b>13</b>	<b>8</b>

**Table VI.1** Analytical results of the parameters found with different methods.

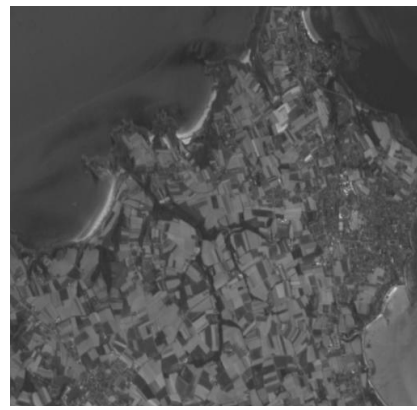
#### ***VI.4.2 Second Experiment***

In the case of important sizes where the data sets are very large and time consuming is very important, we have suggested to employ the NSCT method for extraction of the feature points as cited in section VI.3.1. The NSCT decomposition of images was performed with the following parameters: N=4 levels and L=4 sub-bands at each level; c=1 and the block neighborhood is of size  $w = 32$ .

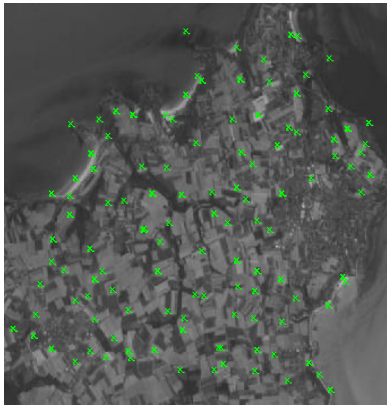
After selecting the corresponding feature points with the NSCT method, we apply the GAs process for registration of the corresponding point sets pair (*Meskine and al., 2012*). The registered image obtained and the pair of images to be compared with its points interest marked with green cross are shown in Fig.VI.6 within a size of 512\*512 pixels.



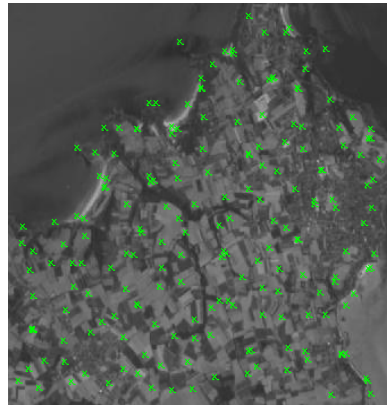
(a)



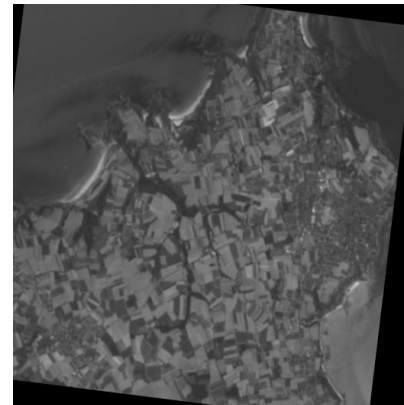
(b)



(c)



(d)



(e)

**Figure IV.6.** Image registration results: (a) reference image, (b) transformed image, (c) feature points extracted from the reference image, (d) feature points extracted from (b), and (e) the resulting image registered with the GA process.

## **VI.5 Results of Application to Medical Images**

In order to demonstrate the capabilities of the proposed algorithm for image registration applied to medical images (*Meskine and al.2012A*). A transformed image is generated from an original image using some known geometrical transformations (translation, rotation).

We have applied the proposed 2D rigid registration algorithm using the NSCT method to generate the data set and a GA as a method of optimization to evaluate the optimal transformation parameters for registering two magnetic resonance (MR) images.

The experiment was conducted using a gray level MR image of size 256\*256. The transformed image to be corrected is rotated by 10 degrees using the bilinear interpolation; and displaced by 15 and -10 pixels in X and Y directions respectively from the center of the reference image.

The simulation results have been obtained using the MATLAB software package. The NSCT decomposition and feature extraction of images are performed according to the following settings:  $N=4$  resolution levels and  $L=4$  directional subbands; the parameter  $c=0.8$  and the block neighbourhood is of size  $w=12$ .

In this case, we have chosen some other parameters for the GA:

Population size: 80 individuals

Crossover probability: 0.85

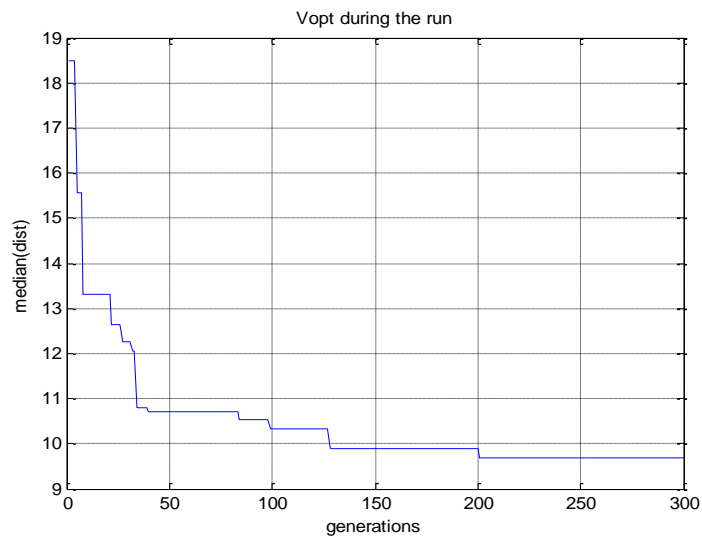
mutation probability: 0.03.

number of generations: 300.

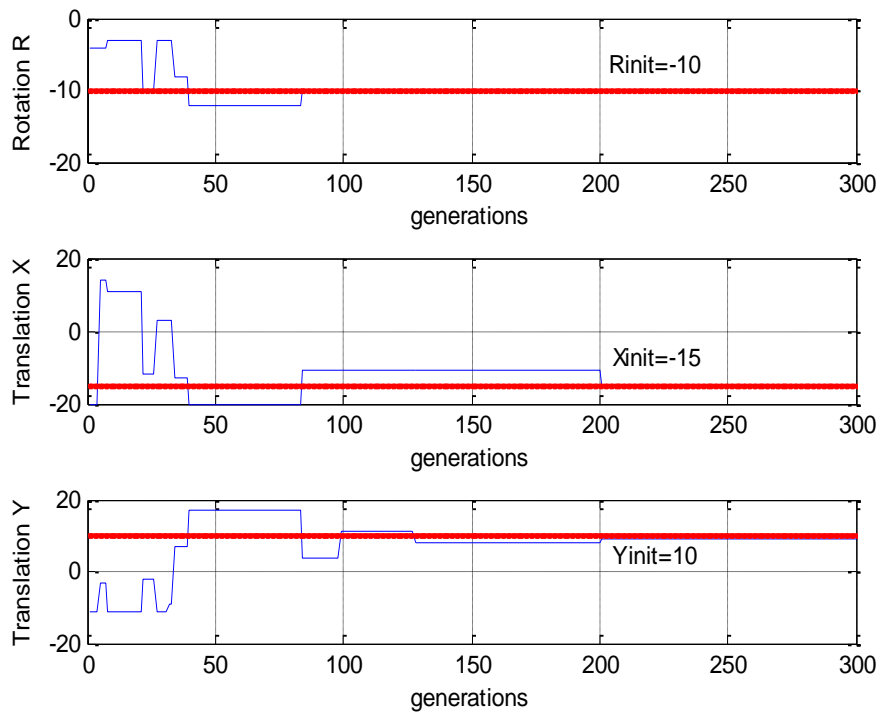
Elitism technique: 5%

The results are shown in Figures VI.7 and VI.8 respectively that illustrate the performance of the algorithm during the GA's run. The results show that the proposed method is very accurate in estimating the rigid transformations. The full time met for reaching the 300 generations is about 425.650235 seconds.





**Figure VI.7.** Evolution of the best solution during the GA process for IRM application.



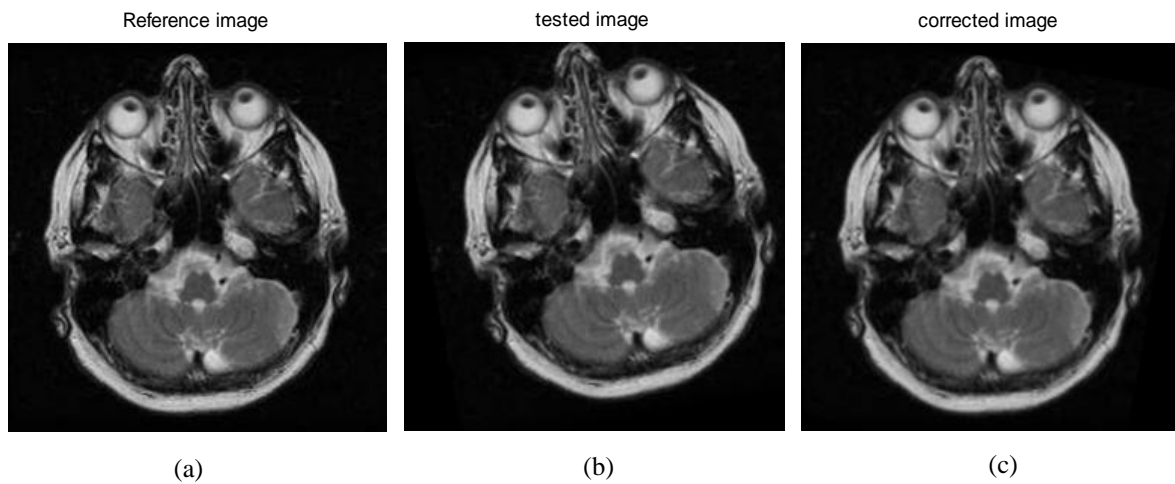
**Figure IV.8.** Evolution of the Transformation parameters R, X and Y during the run of GA in the case of IRM application.

The analytical results are depicted in the following table.

<b>Transformation parameters</b>	<b>R (degrees)</b>	<b>X (pixels)</b>	<b>Y (pixels)</b>
The true parameters	10	15	-10
The obtained parameters	10	15	-9

**Table VI.2.** Comparison of the obtained transformation parameters using the proposed registration algorithm with the true/known ones.

It can be seen from Table VI.2 that the estimated transformation parameters are very close to the true ones. This demonstrates the efficiency and robustness of the proposed registration algorithm. The two images to be aligned and the resulting registered image are shown in Fig.VI.9.



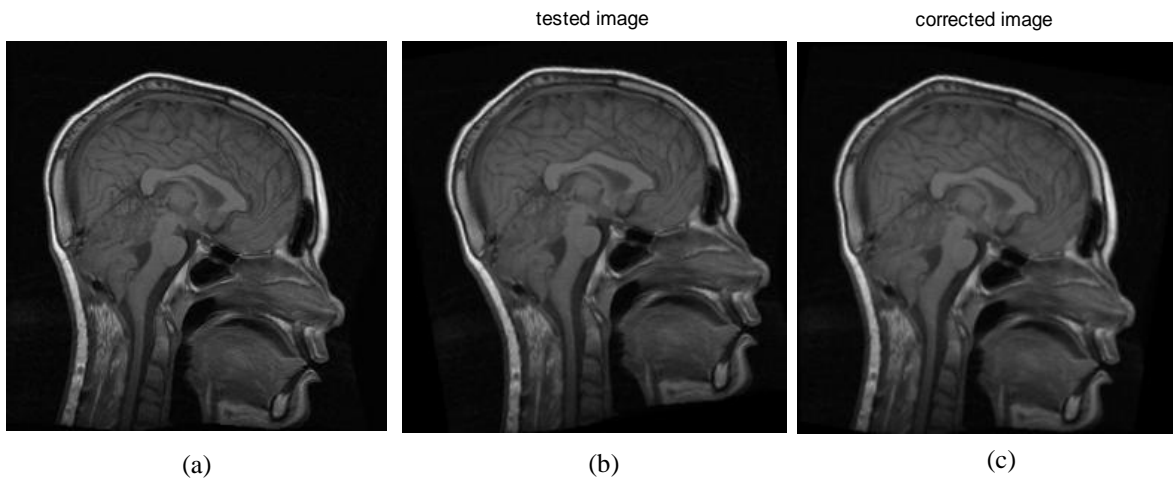
**Figure VI.9.** Results of an MR image registration: (a) Source or reference image, (b) Transformed or tested image, and (c) Registered image with our proposed method of registration based on GAs.

We have also applied our algorithm to register another IRM image (image 2) in which we have applied different tested parameters. The analytical results of this image are depicted in the following table

	Initial parameters			Parameters found		
	R	X	Y	R	X	Y
Test 1	10	15	-10	10	16	10
Test 2	-8	-10	10	-8	-10	-8
Test 3	10	10	-9	9	10	-9

**Table VI.3.** Results of different tests applied for image 2.

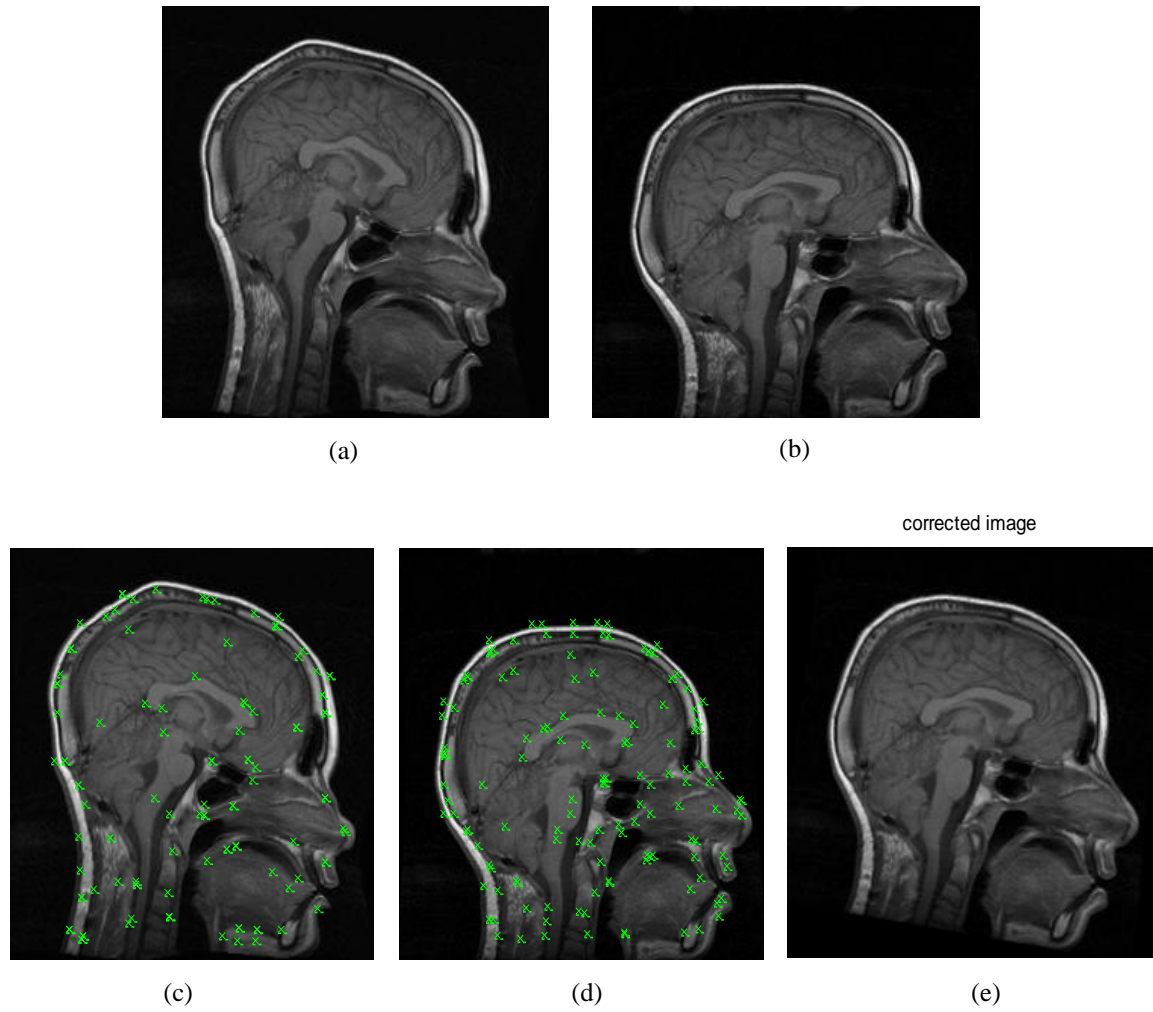
Figure VI.10 shows the registration image results of test 3.



**Figure VI.10.** Registration result of image 2: (a) reference image, (b) tested image, and (c) corrected image with the parameters found with GA.

***Real Application:***

To show the robustness and efficiency of the proposed registration method, we have applied our algorithm to register two different clinical MR images (*Meskine and al., 2012B*). Figure VI.11 shows the resulted registration image using the GAs process with the interest points of both images to be compared.



**Figure VI.11.** Registration results of the two different clinical MR images: (a) source image, (b) target image, (c) source image within feature points extracted, (d) target image to be registered within feature points extracted with NSCT transform, and (e) registered image.

## **VI.6 Conclusion**

Point set registration is among the most fundamental applications in vision research. It is widely used in areas such as range data fusion, medical image alignment, object localization, tracking, object recognition, just to name a few. The goal of the registration task is to find the transformation that best represents the relative transformation between two sets of data. In this chapter, we have introduced a new technique for rigid registration of point sets applied to 2D images. Our interest in this contribution stems from its application in remote sensing as well as medical applications specially for aligning a satellite and IRM images.

The basic procedure involves feature identification, matching of corresponding features, and the alignment of these matches by evaluating a distance. A multi-resolution feature point extraction technique based on the NSCT transform is employed to determine the data set point. Therefore, a GA is performed to estimate the transformation parameters between the two data sets to achieve the image registration task by minimizing the distance between the two sets. In this study, the transformation model employed is the rigid model that we are concentrated on the determination of the translation and rotation displacements. The obtained results are very promising as shown in the simulation, which demonstrate the efficiency and accuracy performance of the proposed method for image registration compared to the intensity image.

To the best of our knowledge from the literature and according to the obtained results, we may state that the 2D-point registration approach is better than the previous proposed methods.

CONCLUSION

## CONCLUSION

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*Registration is an important pre-processing step that enables the use of satellite images for environmental studies and the use of sequences of radiology images for studying the progression of medical pathology.*

*The main obstacle in image registration is the precise estimation of a mapping function that determines the geometric transformation between two image coordinate systems. In this thesis, we have developed a global optimal method in order to get a registration approach with high accuracy based on the application of GAs.*

*GAs represent an intelligent exploitation of a random search used to solve optimization problems. They provide a directed random search in complex landscapes. There are two important issues with respect to search strategies: exploration (investigate new and unknown areas in search space) and exploitation (make use of knowledge of solutions previously found in search space to help in finding better solutions). This can be done by making genetic operators perform essentially a blind search; with a hope that selection operators direct the genetic search toward the desirable area of solution space.*

*Therefore, in this thesis, we have proposed a GA registration algorithm within a hybrid scheme based on the two techniques namely: Fitness sharing and Elitism. The first one keeps the diversity of the population and the second one preserves the best solutions during the run. The algorithm attempts to recover a rigid transformation consisting of three parameters; the rotation and translations in  $x$  and  $y$  directions. The correlation coefficient is used as the objective function of the GA. The algorithm is applied to satellite images and the results are promising.*

*When the number of transformation parameters and the size of the data set increase, the computational cost also increases and the problem becomes very complex. The study of recent multi-resolution search and optimization algorithms applied to image registration has offered new perspectives to handle this challenge. The basic idea is to use the multi-resolution property to produce coarse-to-fine representation of the input images. The search strategy following the multiresolution decomposition works iteratively from the deepest to the top level, going from coarse to fine spatial resolution in order to refine the transformation parameters. Two multiresolution transforms are considered; the wavelet transform and the shift invariant contourlet transform named NSCT.*

## CONCLUSION

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*The main contribution of this thesis is the use of the GAs within a multi-resolution framework based on the NSCT which provides a directional multi-resolution image representation for performing an efficient, robust and accurate rigid registration.*

*An adaptable genetic algorithm for registration is adopted in order to minimize the search space within a hybrid scheme. Two NSCT based methods are proposed for registration. The first one consists of performing the registration for all sub-bands while the second method uses just the LL sub band in order to speed up the search and to minimise the time complexity of the optimization process. The simulation results show the effectiveness of this approach compared to the wavelet method. Moreover, this approach performs better than the NSCT enhancement method. The proposed NSCT registration method has been applied for registering high resolution satellite and radar images, and works perfectly well for multi-temporal images as well, even in the presence of noise.*

*In another contribution, we have introduced a new technique for rigid registration of point sets of 2D images. The basic procedure involves feature identification, matching of correspondent features, and the alignment of these matches by evaluating a distance. A multi-resolution feature points extraction technique based on the NSCT transform is employed to extract the data point set. Therefore, the GA is performed to estimate the transformation parameters between the two data sets generated to achieve the image registration task by minimizing a metric based on the Euclidean distance and ICP concept. The obtained results are very promising as shown in the simulation results compared with the ICP algorithm. To demonstrate the efficiency and accuracy performance of the proposed registration method, we have applied it to register medical MR images and the results are also promising.*

*Further opportunities exist for extending the capability and application of the proposed point registration approach to retinal images. Several enhancements can be explored, in specific, the control point selection and the search space from 2D to 3D which considers the rotation and translation in three axes.*



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